Semantics of Conjectures

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Abstract

This paper aims to expand and detail the notion of formal semantics of *Conjectures* ([Gio21]) by applying the theoretic-model approach seen in [Hay04], [Pat14] and related works.

After a short introduction to the concepts and basics of *Conjectures*, we will start from the notion of Simple Interpretation of RDF, applying and extending the semantic rules and conditions to fully cover the concepts and features of *Conjectures*.

1 Conjectures

The semantics of RDF makes it impossible to express contrasting points of view that act on the same data.

Every approach applied in the past has not solved the main issue, that is, being able to express different statements whose truth value is not known, or even in contrast with each other, without fully asserting them, therefore making them undoubtedly true.

In [Gio21] we can see a novel approach: by using special assertions, called *Conjectures*, we are able to express the statements whose truth value is not given.

The approach revolves around two main concepts:

- *conjecture*: a *concept whose truth value is not available* and its representation;
- *collapse to reality*: a mechanism to fully assert, when the time comes, the truth value of conjectures in their RDF-esque form.

Conjectures encapsulate plain RDF statements or other conjectures in specially marked named graphs. They can be expressed in a strong form:

```
CONJECTURE :deVereWroteHamlet {
    :Hamlet dc:creator :EdwardDeVere .
}
```

or, allowing us to be able to include them in plain RDF 1.1 datasets, in a weak form, where the predicate is expressed as a *conjectural predicate*:

```
@prefix conj0001: <http://example.org/exampleDoc#deVereWroteHamlet>

GRAPH :deVereWroteHamlet {
  :Hamlet conj0001:creator :EdwardDeVere .
  conj0001:creator conj:isAConjecturalFormOf dc:creator .
}
:deVereWroteHamlet prov:wasAttributedTo :JThomasLooney .
```

Additional statements adding information to the conjecture, but external to it, are the *grounds* or *ground statements*. One of them is the last triple in the last example.

More formally, assume the disjoint sets $\mathcal{I}$ (all IRIs), $\mathcal{B}$ (blank nodes), and $\mathcal{L}$ (literals). An RDF triple is a tuple $(s, p, o) \in \mathcal{T} = (\mathcal{I} \cup \mathcal{B}) \times \mathcal{I} \times (\mathcal{I} \cup \mathcal{B} \cup \mathcal{L})$.

For every RDF predicate p, let $\mathcal{S}_p \subseteq (\mathcal{I} \cup \mathcal{B})$ be its domain and $\mathcal{O}_p \subseteq (\mathcal{I} \cup \mathcal{B} \cup \mathcal{L})$ its range.

We denote with

```
x { s p o }
```

a triple (s, p, o) that is referred to in the examples by the name x.

Conjecturing: Conjecturing is the function $conj : \mathcal{T} \to \mathcal{T}$ such that, for every RDF triple $t_1 = (s_1, p, o_1), conj(t_1) = (s, p_{s,o}, o)$ where:

1. Identity of subject: $s_1 = s$.
2. Identity of object: $o_1 = o$.
3. Disjointness: $\forall s_j, o_k$ such that $(s_j, p_{s,o}, o_k) \in \mathcal{T}$, we have that $s = s_j$ and $o = o_k$.

Conjectures, conjectural predicates, conjecturing triple: given the triple $(s, p, o) \in \mathcal{T}$, its conjecture is the triple

$$conj((s, p, o)) = (s, p_{s,o}, o).$$

Conjectural predicates (or *weak predicates*) of predicate p are all the predicates that are members of the set $\mathcal{C}onj_p$, such that:

$$\mathcal{C}onj_p = \{cp \in \mathcal{I} | \exists s \in S_p, o \in O_p, conj((s, p, o)) = (s, cp, o)\}$$

.

Theorem 1: every conjectural predicate is used in one triple only.

Proof (sketch): derives from item 3 of definition 1 (Disjointness): Given two triples $(s_1, p_{s,o}, o_1), (s_2, p_{s,o}, o_2) \in \mathcal{T}$.

For item 3 of definition 1 (Disjointness), we have that $s_1 = s_2$ and $o_1 = o_2 . \forall (s,p,o) \in \mathcal{T}, \exists ! p_{s,o}$ such that $conj((s,p,o)) = (s, p_{s,o}, o)$.

Corollary 1: the function *conj* is unique (barring predicate name changes).

Proof (sketch): derives immediately from Theorem 1.

Conjectural Form: predicate q is said to be a *conjectural form* of p if there exists a pair of subjects and objects s, o such that $conj((s,p,o)) = (s,q,o)$.

Collapsing: Collapsing is a function $collapsesInto : \mathcal{T} \rightarrow \mathcal{T}$ such that, for every RDF triple $t = (s,p,o)$, $collapsesInto(s, p_{s,o}, o) = (s,p,o)$ iff $conj((s,p,o)) = (s, p_{s,o}, o)$, and is undefined otherwise.

Collapsed predicate; collapse: let $(s, p_{s,o}, o)$ be a conjecture. The collapsed predicate of $p_{s,o}$ is the predicate p such that

$$collapsesInto(s, p_{s,o}, o) = (s,p,o)$$

. The collapse is the triple

$$((s, p_{s,o}, o), collapsesInto, (s,p,o))$$

.

2 RDF Simple interpretation and Conjectures I

In RDF, a simple interpretation I of a vocabulary V consists of:

1. A non-empty set IR of resources, called the domain or universe of I.

2. A set IP, called the set of properties of I.

3. A mapping $IEXT$ from IP into the powerset of $IR \times IR$ i.e. the set of sets of pairs $< x, y >$ with x and y in IR .

4. A mapping IS from IRIs into IR - in order to map resources and properties

5. A partial mapping IL from literals into IR - in order to map literals

$IEXT(p)$ is the extension of p, that is the set of pairs that are the arguments for which the property p is true.

According to [Pat14], a semantic extension is a set of additional semantic assumptions that gives IRIs additional meanings on the basis of other specifications or conventions. When this happens, the semantic extension must conform to the minimal truth conditions already enunciated, extending from them to accommodate the additional meanings.

Therefore, we extend the RDF Simple Interpretation adding a new set of conjectural properties, IPC, disjoint from the set of properties IP, where the conjectural predicates are created on the fly.

We add a new mapping $IEXTC$ from IPC to the Cartesian product $IR \times IR$. $IEXTC(cp)$ identifies the pair for which the property cp is true.

Because of the Disjointness property of the conjecturing function $conj$ seen in section 1, the pair satisfying the property cp will always be unique.

We need to specify an additional mapping $CONJFORM$ from IP into IPC to map the conjectural forms of the properties.

Our full simple interpretation of I in RDF with Conjectures is:

1. A non-empty set IR of resources, called the domain or universe of I.

2. A set IP, called the set of properties of I.

3. A set IPC, called the set of conjectural properties of I. $IPC \cap IP = \emptyset$

4. A mapping $IEXT$ from IP into the powerset of $IR \times IR$ i.e. the set of sets of pairs $< x, y >$ with $x, y \in IR$.

5. An injective mapping $IEXTC$ from IPC into $IR \times IR$, in other words the set of pairs $< x, y >$ with $x, y \in IR$. By definition of injective mapping, if $IEXTC(a) = IEXTC(b)$, then $a = b$, that is, any $cp \in IPC$ uniquely applies to a specific pair $< x, y >$.

6. A mapping $CONJFORM$ from IP into IPC in order to map the conjectural forms of the properties

7. A mapping IS from IRIs into IR - in order to map resources, properties and conjectural properties

8. A partial mapping IL from literals into IR - in order to map literals

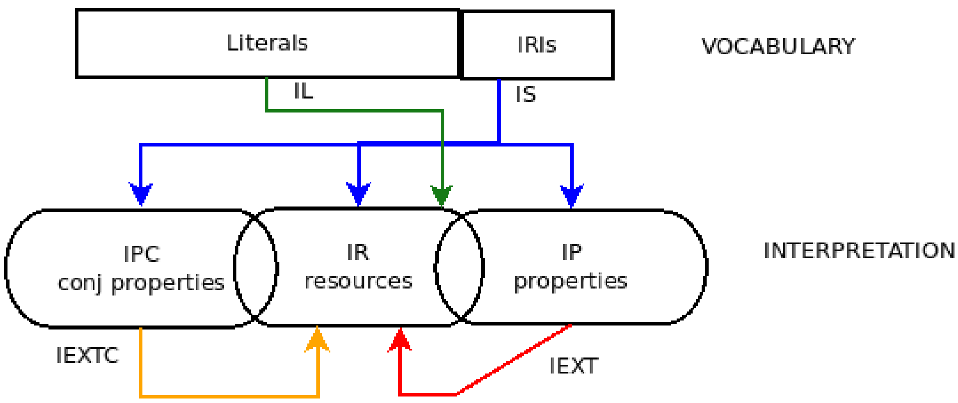

Figure 1: Schematics of the interpretation

3 Semantic conditions for ground graphs with Conjectures

- if E is a literal then $I(E) = IL(E)$
- if E is an IRI then $I(E) = IS(E)$
- if E is a ground triple $s\ p\ o$. then
  - $I(E) = true$ if $I(p) \in IP$ and
  - the pair $< I(s), I(o) >\in IEXT(I(p))$
  - otherwise $I(E) = false$.
- if E is a Conjecture triple $s\ cp\ o$. then
  - $I(E) = true$ if $I(cp) \in IPC$ and
  - the pair $< I(s), I(o) >\in IEXTC(I(cp))$
  - $I(cp) \in CONJFORM(I(p))$ for some $p \in IP$
  - otherwise $I(E) = false$.
- if E is a ground RDF graph then $I(E) = false$ if $I(E') = false$ for some triple $E' \in E$, otherwise $I(E) = true$.
- if E is a ground conjectural graph then $I(E) = false$ if $I(E') = false$ for some triple $E' \in E$, otherwise $I(E) = true$.

The last clause captures the definition of conjectural graph, that is the representation of a conjecture as a list of individual statements composing as a whole the conjecture.

3.1 Model

Given a RDF graph G, we say that the interpretation I is a model of the graph G if all the triples of graph G are satisfied, in other words, Interpretation I satisfies G.

The notion of model is at the basis of the (simple) entailment: following standard terminology, we say that I (simply) satisfies G when $I(G) = true$, that G is (simply) satisfiable when a simple interpretation exists which satisfies it, otherwise (simply) unsatisfiable, and that a graph E simply entails a graph G when every interpretation which satisfies E also satisfies G. If two graphs G and F each entail the other then they are logically equivalent. [Pat14]

Is our interpretation I a model of this conjecture graph?

```
:deVereWroteHamlet {
   :Hamlet conj001:creator :EdwardDeVere .
   conj001:creator conj:isAConjecturalFormOf dc:creator .
}
```

- $IR = \{dVWH, h, c, cc1, e, iacf\}$
- $IP = \{c, iacf\}$
- $IPC = \{cc1\}$

The functions:

- $IS(:deVereWroteHamlet) \rightarrow dVWH$
- $IS(:Hamlet) \rightarrow h$
- $IS(conj001:creator) \rightarrow cc1$

- $IS(:EdwardDeVere) \rightarrow e$
- $IS(conj : isAConjecturalFormOf) \rightarrow iacf$
- $IS(dc : creator) \rightarrow c$
- $IEXT(c) = \emptyset$
- $IEXT(iacf) = \{< cc1, c >\}$
- $IEXTC(cc1) = \{< h, e >\}$
- $CONJFORM(c) = cc1$

IEXT(c) is the empty set. We are still dealing with a conjecture, there is no assertion regarding any "real" property (in this case $dc : creator$) yet.

All the clauses seem to hold. We can say that simple interpretation (s-interpretation) I is a model of our graph.

4 Blank Nodes

With conjectures, we would want to be able to express even more "uncertain" concepts involving unnamed entities or unspecified values.

In RDF this is done through *blank nodes*, which indicate the existence of an entity without using a IRI to identify any particular one.

In this section we will use a simple sentence implying the reliance on a blank node, namely "Muammar al-Qaddafi claimed that the author of Othello was someone who was an Arab" [1].

In this case we don't know who someone is, and we can't identify it with any IRIs.

[1] He really did it in December 1988 - see "William Shakespeare's Othello" by Jibesh Bhattacharyya, ISBN 9788126904747

Nevertheless, the information we are conveying with our conjecture maintains some degree of meaningfulness, and it definitely is something we could reason upon.

Sticking to our examples' style, we could say it like this: "it is conjectured that Othello was written by somebody and this somebody was an Arab. And this claim has been attributed to Muammar al-Qaddafi".

In RDF:

```
:ArabWroteOthello {
  :Othello conj002:creator _:z .
  conj002:creator conj:isAConjecturalFormOf dc:creator .
  _:z dbpedia-owl:nationality :Arab .}


:ArabWroteOthello prov:wasAttributedTo :MalQaddafi .
```

It comes pretty natural to conceive the term "somebody" as a blank node.

4.1 Interpretation $[I + A]$

In order to deal with blank nodes, we need to add a new mapping A from blank nodes into IR.

Therefore, we extend our interpretation I:

- $[I + A](x) = I(x)$ for names
- $[I + A](x) = A(x)$ if x is a blank node.

We add a couple of semantic conditions to our interpretation for blank nodes, one is the "standard" one for RDF graphs, the other one is for conjectures:

- if E is a ground RDF graph then $I(E) = true$ if $[I + A](E) = true$ for some mapping A from the set of blank nodes in E to IR, otherwise $I(E) = false$.

- if E is a ground conjectural graph then $I(E) = true$ if $[I+A](E) = true$ for some mapping A from the set of blank nodes in E to IR, otherwise $I(E) = false$.

4.2 Model

Is our Interpretation $[I+A]$ with blank nodes a model of our example graph?

Let's reason on $: ArabWroteOthello$ part only.

Be A our blank nodes mapping into IR: $\_ : z \rightarrow zz$.

Our interpretation $[I + A]$ will be:

- $IR = \{awo, o, c, cc2, iacf, n, a, zz\}$

- $IP = \{c, iacf, n\}$

- $IPC = \{cc2\}$

The functions:

- $IS(: ArabWroteOthello) \rightarrow awo$

- $IS(: Othello) \rightarrow o$

- $IS(conj002 : creator) \rightarrow cc2$

- $IS(dc : creator) \rightarrow c$

- $IS(dbpedia - owl : nationality) \rightarrow n$

- $IS(: Arab) \rightarrow a$

- $IEXT(c) = \emptyset$
- $IEXT(iacf) = \{< cc2, c >\}$
- $CONJFORM(c) = cc2$
- $IEXT(n) = \{< zz, a >\}$
- $IEXTC(cc2) = \{< o, zz >\}$

Even in this case $IEXT(c)$ is empty because there is no assertion regarding the property $dc : creator$ yet, so our conjecture is, well, still a conjecture.

The last two functions hold because of the mapping A of our blank node.

All the clauses are true. Our interpretation $I + A$ is a model of our graph.

This approach allows us to reason with the blank nodes in what-if scenarios, where we could define the A mapping from blank nodes to specific IRIs of our choice, therefore exploring new relationships arising between the triples.

5 Collapse to reality

At a certain point, someone may want to consider our conjectures as true.

In this case we would need to specify a mechanism in order to express the statements in the conjecture in full force.

This mechanism is called "collapse to reality".

It consists of exactly two triples added to the base, where:

- in the first new triple, the conjectural property is used instead of the "effective" property it is a conjectural form of.

- the conjecture is declare to "collapse into" the first new triple.

Two new triples are added, nothing gets replaced or deleted. So we can keep track of what's happening and all the relationships between the graphs.

Let's reason on an example of a collapse:

```
:attr1 {
    :Hamlet conj003:creator :Shakespeare .
    conj003:creator conj:isAConjecturalFormOf dc:creator.
}

:attr1Cot {
    :Hamlet dc:creator :Shakespeare .
}

:attr1  conj:collapsesInto :attr1Cot .
```

$: attr1$ is collapsed by adding a triple $: attr1Cot$ where the conjectural predicate is replaced by the corresponding "real" predicate.

The final additional triple functions as the explication of the collapse. The conjecture $: attr1$ is declared to be "collapsing" into $: attr1Cot$ by means of the property "conj:collapsesInto".

For the seek of clarity:

- the conjecture triple to be collapsed will be the "conjecture triple";
- the new triple collapsing the conjecture will be the "collapsing triple";
- the last triple will be the "collapsesInto triple".

5.1 Interpretation $[I + COLLAPSE]$

We need to extend once more our Interpretation with Conjectures I by adding the Collapse to Reality rules.

We define a new mapping $collapsesInto$ from triples to triples that maps the relationship between the conjecture triple and its collapsing triple:

$collapsesInto(s, cp, o) = (s, p, o)$ iff $conj(s, p, o) = (s, cp, o)$.

The conjecture triple is allowed to collapse into the collapsing triple if, and only if, it itself is the (unique) conjecture of the collapsing triple.

We can clearly see that the "collapsesInto triple" is just the translation into RDF of the definition of collapse we have introduced in section 1, that is, the triple:

$$((s, cp, o), \quad collapsesInto, \quad (s, p, o))$$

From a more formal point of view, given a conjectural triple E, we add the triple E_{cot} as a collapsing triple $\{s \quad p \quad o.\}$.

The semantic conditions of the collapse to reality should be the following:

- let E be a conjecture triple

  $\{s \quad cp \quad o \quad .\}$

  let E_{cot} be a collapsing triple

  $\{s \quad p \quad o \quad .\}$

  and finally let $E_{collapsesInto}$ be the collapsesInto triple

  $\{E \quad \mathit{collapsesInto} \quad E_{cot}\}$

  then

  - $I(E) = true$ if $I(cp) \in IPC$ and
  - $I(E_{cot}) = true$ if $I(p) \in IP$ and

- $CONJFORM(I(p)) = I(cp)$ and
- the pair $< I(s), I(o) >\in IEXTC(I(cp))$ and
- the pair $< I(s), I(o) >\in IEXT(I(p))$ and
- $I(E_{collapsesInto}) = true$ if $collapsesInto(I(E)) = I(E_{cot})$, that is: $collapsesInto(I(s), I(cp), I(o)) = (I(s), I(p), I(o))$
- otherwise $I(E) = false$ and $I(E_{cot}) = false$ and $I(E_{collapseInto}) = false$.

5.2 Model

Now, let's check if our Interpretation $[I + COLLAPSE]$ can be a model of our example graph.

- $IR = \{a1, h, cc3, s, iacf, c, a1cot, ci\}$
- $IP = \{iacf, c, ci\}$
- $IPC = \{cc3\}$

The functions:

- $IS(: attr1) \to a1$
- $IS(: Hamlet) \to h$
- $IS(conj003 : creator) \to cc3$
- $IS(: Shakespeare) \to s$
- $IS(conj : isAConjecturalFormOf) \to iacf$
- $IS(dc : creator) \to c$
- $IS(: attr1Cot) \to a1cot$

- $IS(conj : collapsesInto) \to ci$
- $IEXT(iacf) = \{< cc3, c >\}$
- $IEXT(c) = \{< h, s >\}$
- $IEXT(ci) = \{< a1, a1cot >\}$
- $IEXTC(cc3) = \{< h, s >\}$
- $CONJFORM(c) = cc3$

Let's check our semantic conditions for the collapse to reality:

- $I(cc3) \in IPC$? Yes;
- $I(c) \in IP$? Yes;
- $CONJFORM(c) = cc3$? Yes;
- The pair $< h, s > \in IEXTC(cc3)$? Yes;
- The pair $< h, s > \in IEXTC(c)$? Yes;
- $collapsesInto(h, cc3, s)) = (h, c, s)$? Yes, because $CONJFORM(c) = cc3$, hence $conj(h, c, s) = (h, cc3, s)$

All of them seem to hold.

Hence, our interpretation $[I + COLLAPSE]$ is a model of our graph.

6 Further applications of conjectures

6.1 Conjecture of a conjecture - nested conjectures

Let's imagine we want to express a conjecture on another conjecture. Of course, the process can involve as many conjectures over conjectures we want.

Such as:

```
:conjecture01 {
    :Hamlet conj004:creator :EdwardDeVere .
    conj004:creator conj:isAConjecturalFormOf dc:creator .
}

:conjecture02 {
 :conjecture01 conj004:wasAttributedTo :JThomasLooney .
 conj004:wasAttributedTo conj:isAConjecturalFormOf prov:wasAttributedTo .
}

:conjecture03 {
:conjecture02 conj004:wasInformedBy <https://bit.ly/3wSH76A> .
conj004:wasInformedBy conj:isAConjecturalFormOf prov:wasInformedBy .
}

:conjecture03 prov:wasAttributedTo :FabioVitali .
```

They are three different conjectures, one becoming the subject of the other one:

- the first one says that Hamlet is conjectured to have been written by Edward De Vere;
- the second one says that the previous conjecture could possibly have been made by J. Thomas Looney;
- The third one says that it might be that the second conjecture could have been brought to light by the resource with URL https://bit.ly/3wSH76A.

- The fourth one says that the previous conjecture has been attributed to Fabio Vitali.

Reading it (more or less) backwards: Fabio Vitali has stated that the resource at https://bit.ly/3wSH76A might have brought to light that J. Thomas Looney could have possibly said that Hamlet is conjectured to have been written by Edward De Vere.

We could imagine it as a stack, or a stair, where at level 0 we have the first conjecture, and then as we get on the higher steps, the conjectures we find are built with the conjecture at the level below as a subject (or object, or both):

```
Level 2:            E{3}={E{2}, cp3, o3}
Level 1:                  E{2}=(E{1}, cp2, o2}
Level 0:                        E{1}=(s1, cp1, o1)
```

Developing the Level 2 we see they are nested in each other: $E3 = (E1, cp2, o2), cp3, o3) = (((s1, cp1, o1), cp2, o2), cp3, o3)$

6.1.1 Interpretation $[I + NESTEDCONJ]$

In order to delineate the rules for the conjectures of conjectures (or nested conjectures), we can reason with pairs of conjectures.

As stated before, the conjectures at levels > 0 could be based on lower-level conjectures as their subject or object, or both.

For the sake of conciseness, we momentarily limit ourselves to reason with the case of the lower-level conjectures as their subject only.

1. if E_0 is a ground conjecture (s_0, cp_0, o_0), $E_1 = (E_0, cp_1, o_1)$
2. if E_1 is a ground conjecture (E_0, cp_1, o_1), $E_2 = (E1, cp_2, o_2)$ [...]

3. if E_{k-1} is a "higher level" ground conjecture $(E_{k-2}, cp_{k-1}, o_{k-1})$, $E_k) = (E_{k-1}, cp_k, o_k)$

4. if E_k is a "higher level" ground conjecture (E_{k-1}, cp_k, o_k), $E_{k+1} = (E_k, cp_{k+1}, o_{k+1})$

We should have the following cases:

- base cases:
  - $E_1 = (E_0, cp_1, o_1)$
  - $E_1 = (s_1, cp_1, E_0)$
- k^{th} cases:
  - $E_k = (E_{k-1}, cp_k, o_k)$
  - $E_k = (s_k, cp_k, E_{k-1})$

Let's extend our interpretation I adding new rules.

The extension will be subdivided into cases, depending on the type of conjectures to be evaluated.

We must also enforce an order on the operations: we start from the conjecture(s) at the "lowest level", that is, the one(s) not involving other conjectures, evaluate them and "climb" the "stair".

Therefore, the extension to the interpretation I will be:

Base case - for the lowest level conjecture at level 0:

- let E_0 be a conjecture triple
  $\{s_0 \quad cp_0 \quad o_0 \quad .\}$ then
  - $I(E_0) = true$ if $I(cp_0) \in IPC$ and
  - the pair $< I(s_0), I(o_0) > \in IEXTC(I(cp_0))$

– otherwise $I(E_0) = false$

As we "climb" up the "stair" and get to the conjecture E_k, we can assume that the conjecture E_{k-1} has already been proven by the previous steps.

K^{th} **Case S** - for conjectures at the first step and above having another conjecture as the subject:

- let E_{k-1} be a conjecture triple

  let E_k be a conjecture triple

  $\{E_{k-1} \quad cp_k \quad o_k \quad .\}$

  then

  – $I(E_k) = true$ if $I(E_{k-1}) = true$ and

  – $I(cp_k) \in IPC$ and

  – the pair $< I(E_{k-1}), I(o_k) >\in IEXTC(I(cp_k))$

  – otherwise $I(E_k) = false$

K^{th} **Case O** - for conjectures at the first step and above having another conjecture as the object:

- let E_{k-1} be a conjecture triple

  let E_k be a conjecture triple

  $\{s_k \quad cp_k \quad E_{k-1} \quad .\}$

  then

  – $I(E_k) = true$ if $I(E_{k-1}) = true$ and

  – $I(cp_k) \in IPC$ and

– the pair $< I(s_k), I(E_{k-1}) > \in IEXTC(I(cp_k))$

– otherwise $I(E_k) = false$

K^{th} **Case SO** - for conjectures at the first step and above having another conjecture as the subject and yet another one as the object:

- let $E_{(k-1)a}$ be a conjecture triple

  let $E_{(k-1)b}$ be a conjecture triple

  let E_k be a conjecture triple

  $\{E_{(k-1)a} \quad cp_k \quad E_{(k-1)b} \quad .\}$

  then

  – $I(E_k) = true$ if $I(E_{(k-1)a}) = true$ and

  – $I(E_{(k-1)b}) = true$ and

  – $I(cp_k) \in IPC$ and

  – the pair $< I(E_{(k-1)a}), I(E_{(k-1)b}) > \in IEXTC(I(cp_k))$

  – otherwise $I(E_k) = false$

6.1.2 Model

Is our Interpretation $[I + NESTEDCONJ]$ a model of the nested conjectures example seen before?

We define the sets:

- $IR = \{c1, h, cc4, edv, iacf, c, c2, cwa4, jtl, pwa, c3, cwib4, http, pwib, fv\}$
- $IP = \{c, iacf, pwa, pwib\}$
- $IPC = \{cc4, cwa4, cwib4\}$

The functions:

- $IS(: conjecture01) \rightarrow c1$
- $IS(: Hamlet) \rightarrow h$
- $IS(conj004 : creator) \rightarrow cc4$
- $IS(: EdwardDeVere) \rightarrow edv$
- $IS(conj : isAConjecturalFormOf) \rightarrow iacf$
- $IS(dc : creator) \rightarrow c$
- $IS(: conjecture02) \rightarrow c2$
- $IS(conj004 : wasAttributedTo) \rightarrow cwa4$
- $IS(: JThomasLooney) \rightarrow jtl$
- $IS(prov : wasAttributedTo) \rightarrow pwa$
- $IS(: conjecture03) \rightarrow c3$
- $IS(conj004 : wasInformedBy) \rightarrow cwib4$
- $IL(< https : //bit.ly/3wSH76A >) \rightarrow http$
- $IS(prov : wasInformedBy) \rightarrow pwib$
- $IS(: FabioVitali) \rightarrow fv$
- $IEXT(iacf) = \{< cc4, c >, < cwa4, pwa >, < cwib4, pwib >\}$
- $IEXT(c) = \emptyset$
- $IEXT(pwa) = \{< c3, fv >\}$
- $IEXT(pwib) = \emptyset$
- $IEXTC(cc4) = \{< h, ev >\}$

- $IEXTC(cwa4) = \{< c1, jtl >\}$
- $IEXTC(cwib4) = \{< c2, http >\}$
- $CONJFORM(c) = cc4$
- $CONJFORM(cwa4) = pwa$
- $CONJFORM(cwib4) = pwib$

Let's check the validity of the rules of the new semantic extension.

We must start from the conjecture not depending on other conjecture, that is : $conjecture01$, and we use the Base Case:

- Is $cc4 \in IPC$? Yes
- Is the pair $< h, ev) >\in IEXTC(cc4)$ Yes

The "base" conjecture seems to hold. We can say that $c1 = true$.

Now we "climb the ladder" to : $conjecture02$. Since it is based on another conjecture (: $conjecture01$, already proved true) as its subject, we use "k^{th} Case S"

- is $c1 = true$? Yes
- is $cwa4 \in IPC$? Yes
- is the pair $< c1, jtl >\in IEXTC(cwa4)$? Yes

Then we can say that $c2 = true$.

Let's climb one step higher. : $conjecture03$ is based on : $conjecture02$ as its subject. We still use "k^{th} Case S"

- is $c2 = true$? Yes
- is $cwib4 \in IPC$? Yes
- is the pair $< c2, http > \in IEXTC(cwib4)$? Yes

$\rightarrow c3 = true$.

The last triple : $conjecture03prov : wasAttributedTo : FabioVitali$ is satisfied by the rules of the simple intepretation I

Everything seems to hold.

We can say that our Interpretation $[I + NESTEDCONJ]$ satisfies all the triples of the graph.

6.2 Conjecture of a collapse

Let E be a conjecture. What if we conjectured its collapse?

Let's reason on this example of a collapse:

```
:attr1 {
    :Hamlet conj005:creator :Shakespeare .
    conj005:creator conj:isAConjecturalFormOf dc:creator .
}

:attr1Cot {
    :Hamlet dc:creator :Shakespeare .
}

:attr1Ci {
    :attr1 conj:collapsesInto attr1Cot .
}
```

where we have that $:attr1$ is the conjecture, $:attr1Cot$ is the *collapsing triple*, $:attr1Ci$ is the *collapsesInto triple*. The collapse states that our conjecture has been declared "effective" and turned into a statement asserting that Hamlet has actually been created by Shakespeare.

Now, we would like to conjecture this collapse, basically saying something like "it's been conjectured that Hamlet could actually have been created by Shakespeare", in other (a bit more formal) words: "it's been conjectured that $:collapseOfAttr1$ MAY collapse $:attr1$".

At the beginning we have just the conjecture E, not collapsed yet:

```
:attr1 {
    :Hamlet conj005:creator :Shakespeare .
    conj005:creator conj:isAConjecturalFormOf dc:creator .
}
```

If we proceeded with the collapse, we would have the triple E_{cot}, that is the collapsing triple, where we declare the validity of the "real" predicate.

Then, we would add the *collapsesInto triple* $E_{collapsesInto}$, where the conjecture and its collapse are linked together.

We start conjecturing the *collapsesInto triple* $E_{collapsesInto}$.

As we have seen in section 1, Given the triple $(s, p, o) \in \mathcal{T}$, its conjecture is the triple $conj((s, p, o)) = (s, p_{s,o}, o)$.

We need to:

- define a new conjectural predicate *cp* such that $CONJFORM(conj : collapsesInto) = cp$. We pick $maybe : collapsesInto$;
- forge on-the-fly a new IRI that will identify the collapsing triple E_{cot}, this for the triple to be well formed. Let's pick $:attr1Cot$.

Our conjecture of the *collapsesInto triple* then will be:

```
:conjOfC1 {
:attr1 maybe:collapsesInto :attr1Cot  .
maybe:collapsesInto conj:isAConjecturalFormOf conj:collapsesInto .
}
```

$: attr1Cot$, a *collapsing triple*, is not defined yet. This is correct, it must not be defined, otherwise it would mean that its content is the real one.

Our full conjecture of the collapse will just be:

```
:attr1 {
    :Hamlet conj005:creator :Shakespeare .
    conj005:creator conj:isAConjecturalFormOf dc:creator .
}


:conjOfCi1 {
:attr1 maybe:collapsesInto :attr1Cot .
maybe:collapsesInto conj:isAConjecturalFormOf conj:collapsesInto .
}
```

6.2.1 Interpretation $[I + COLLAPSE + CONJOFCOLLAPSE]$

We need to extend the "Simple Interpretation with Conjectures and Collapse" we have defined in section 5.1 in order to accommodate the conjecture of a collapse.

We define a mapping $CONJ$ from triples to triples such that $conj((s,p,o)) = (s, p_{s,o}, o)$ where $CONJFORM(p) = p_{s,o}$.

And, for the first time, we must add a condition on a specific IRI, that is $conj : collapsesInto$: it must be recognized by all the interpretations.

- let E be a conjecture triple

  $\{s \quad cp \quad o \quad .\}$

  let $E_{conjcollapseInto}$ be the conjecture of the collapse

  $\{E \quad mc \quad E_{cot}\}$

  then

  - $I(E) = true$ if $I(cp) \in IPC$ and
  - $I(E_{conjcollapseInto}) = true$ if $CONJ((I(E), I(conj : collapsesInto), I(E_{cot}))) = I(E_{conjcollapseInto})$, therefore $CONJFORM(I(conj : collapsesInto)) = I(mc)$ and
  - the pair $< I(s), I(o) > \in IEXTC(I(cp))$ and
  - the pair $< I(E), I(E_{cot}) > \in IEXTC(I(mc))$
  - otherwise $I(E) = false$ and $I(E_{cot}) = false$ and $I(E_{collapse}) = false$.

6.2.2 Model

Is our Interpretation $[I + COLLAPSE + CONJOFCOLLAPSE]$ a model of the example of a conjecture of a collapse seen before?

We define the sets:

- $IR = \{a1, h, cc5, s, iacf, c, coc1, mc, a1c, ci\}$
- $IP = \{c, iacf\}$
- $IPC = \{cc5, mc\}$

The functions:

- $IS(: attr1) \rightarrow a1$

- $IS(:Hamlet) \to h$
- $IS(conj005:creator) \to cc5$
- $IS(:Shakespeare) \to s$
- $IS(conj:isAConjecturalFormOf) \to iacf$
- $IS(dc:creator) \to c$
- $IS(:conjOfCi1) \to coc1$
- $IS(:attr1Cot) \to a1c$
- $IS(maybe:collapsesInto) \to mc$
- $IS(conj:collapsesInto) \to ci$
- $IEXT(c) = \emptyset$
- $IEXT(iacf) = \{< cc5, c >, < mc, ci >\}$
- $IEXTC(cc5) = \{< h, s >\}$
- $IEXTC(mc) = \{< a1, a1c >\}$
- $CONJFORM(c) = cc5$
- $CONJFORM(ci) = mc$

Let's check the validity of the rules of the new semantic extension $CONJOFCOLLAPSE$:

- $cc5 \in IPC$? Yes.
- $CONJ((a1, ci, a1c)) = (a1, mc, a1c))$ therefore $CONJFORM(I(ci)) = I(mc)$? Yes, that holds
- the pair $< h, s > \in IEXTC(cc5)$? Yes.

- the pair $< a1, a1c > \in IEXTC(mc)$ Yes.

Everything seems to hold.

We can say that our Interpretation $[I{+}COLLAPSE{+}CONJOFCOLLAPSE]$ satisfies all the triples of the graph.

6.3 Cascading Collapses

As we have seen before, a collapse of a conjecture is composed of the conjecture plus two triples:

- the collapsing triple
- the collapse triple

Now we want to cascading-collapse our conjecture of the collapse seen before. We add the additional two triples. $: cot1$ will be our **collapsing triple** and $: collapsesInto1$ will be our **collapsesInto triple**

```
:attr1 {
    :Hamlet conj006:creator :Shakespeare .
    conj006:creator conj:isAConjecturalFormOf dc:creator .
}

:conjOfCi1 {
:attr1 maybe:collapsesInto attr1Cot  .
maybe:collapsesInto conj:isAConjecturalFormOf conj:collapsesInto .
}

:conjOfCi1cot{
    :attr1 conj:collapsesInto :attr1Cot .
```

```
}

:conjOfCi1Ci: {
    :conjOfCi1 conj:collapsesInto :conjOfCi1cot .
}
```

Now we know that $:attr1$ actually collapses into $:attr1Cot$. So, let's go ahead and collapse the remainder of the collapses of the graph, that is $:attr1$. We add two triples: $attr1Cot$ will be the *collapsing triple* while $attr1Ci$ will be the *collapseInto triple*:

```
:attr1 {
    :Hamlet conj006:creator :Shakespeare .
    conj006:creator conj:isAConjecturalFormOf dc:creator .
}

:attr1Cot {
    :Hamlet dc:creator :Shakespeare .
}

:attr1Ci {
    :attr1 conj:collapsesInto :attr1Cot .
}

:conjOfCi1 {
:attr1 maybe:collapsesInto attr1Cot  .
maybe:collapsesInto conj:isAConjecturalFormOf conj:collapsesInto .
}
```

```
:conjOfC1Cot{
    :attr1 conj:collapsesInto :attr1Cot .
}

:conjOfC1Ci: {
    :conjOfCi1 conj:collapsesInto :conjOfC1Cot .
}
```

Interesting fact: we have $:conjOfC1Cot$, generated from the collapse of the conjecture of the collapse, that is equivalent to $attr1Ci$, generated from the collapse of the conjecture $:attr1$, and this connects the two collapses in a way.

6.3.1 Model

Now, let's check if the Interpretation $[I{+}COLLAPSE{+}CONJOFCOLLAPSE]$ we have developed in section 6.2.1 can be a model of our example graph, too.

The sets:

- $IR = \{a1, h, cc6, s, iacf, c, a1c, a1ci, ci, cc1, mc, cc1cot, cc1ci\}$
- $IP = \{c, iacf, ci\}$
- $IPC = \{cc6, mc\}$

The functions:

- $IS(: attr1) \rightarrow a1$

- $IS(: Hamlet) \to h$
- $IS(conj006 : creator) \to cc6$
- $IS(: Shakespeare) \to s$
- $IS(conj : isAConjecturalFormOf) \to iacf$
- $IS(dc : creator) \to c$
- $IS(: attr1Cot) \to a1c$
- $IS(: attr1Ci) \to a1ci2$
- $IS(conj : collapsesInto) \to ci$
- $IS(: conjOfCi1) \to cc1$
- $IS(maybe : collapsesInto) \to mc$
- $IS(: conjOfC1Cot) \to cc1cot$
- $IS(: conjOfC1Ci) \to cc1ci$
- $IEXT(c) = \{< h, s >\}$
- $IEXT(iacf) = \{< cc6, c >, < mc, ci >\}$
- $IEXT(ci) = \{< a1, a1c >\}$
- $IEXTC(cc6) = \{< h, s >\}$
- $IEXTC(mc) = \{< a1, a1c >\}$
- $CONJFORM(c) = cc6$
- $CONJFORM(ci) = mc$

Let's check the validity of the rules of the semantic extension $CONJOFCOLLAPSE$:

- $I(conj006 : creator) \rightarrow cc6 \in IPC$? Yes.
- $CONJ((a1, ci, a1c)) = (a1, mc, a1c))$, therefore $CONJFORM(I(ci)) = I(mc)$? Yes, that holds
- the pair $< h, s > \in IEXTC(cc6)$? Yes.
- the pair $< sh, a1 > \in IEXTC(I(mc))$ Yes.

Let's check the validity of the rules of the collapses $[I + COLLAPSE]$. We have two collapses.

The collapse of the conjecture : $attr1$ has already been proved against in section 5.

Regarding our *conjecture of a collapse* : $conjOfCi1$:

- $I(cc1) = true$ if $mc \in IPC$: Yes, it holds
- $I(cc1cot) = true$ if $ci \in IP$: Yes, it holds
- $CONJFORM(ci) = mc$? Yes
- the pair $< a1, a1c > \in IEXTC(mc)$? Yes
- the pair $< a1, a1c > \in IEXT(ci)$? Yes
- $I(cc1ci) = true$ if $collapsesInto(cc1) = cc1cot$, that is: $collapsesInto(a1, mc, a1c) = (a1, ci, a1c)$ Does it hold? Yes, because $CONJFORM(ci) = mc$, hence $conj(a1, ci, a1c) = (a1, mc, a1c)$

It all holds. Our interpretation is a model.

7 Conclusions

We have tried to delineate a formal semantics for the *Conjectures*.

This work is not intended to be complete, rather it is meant to act as a mere starting point for a proper sound formal model for the *Conjectures*.

References

[Hay04] Patrick Hayes. *RDF Semantics*. 2004. URL: `https://www.w3.org/TR/rdf-mt/`.

[Pat14] Peter F. Patel-Schneider Patrick J. Hayes. *RDF 1.1 Semantics*. 2014. URL: `https://www.w3.org/TR/rdf11-mt/`.

[Gio21] Fabio Vitali Gioele Barabucci Francesca Tomasi. "Expressing RDF statements for conjectural, uncertain, hypothetical and dubious facts in cultural heritage". In: (2021).