# An OWL Full Interpretation

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### Abstract

This report is an appendix to report HPL-2008-59. It gives a worked example of the construction used in the proof from that report. For finiteness, a reduced datatype map consisting of only xsd:boolean is used. Each of the graphs in the construction is listed explicitly, with some redundancy eliminated. The final Herbrand graph contains about 15,000 triples.

### 1 Introduction

This is an appendix to [1, 2]. The construction in those reports builds a Herbrand graph in which the property extension of every property is totally explicit. This appendix presents a complete worked example.

The construction is essentially an infinite one. We make it finite in the following ways:

- We have a very small datatype map D, being { xsd:boolean }
- We pretend  $L_{\text{plain}} = \{""\}$
- We use the ter Horst technique of ignoring every rdf: *i* except the first; these only appear because of their presence in the RDF and RDFS axioms.

The first of these two are non-conformant. Datatype maps are required to include xsd:string, xsd:integer and XMLLiteral.  $L_{plain}$  is infinite. The differences are of no great consequence here.

Each section presents one of the graphs in the construction of section 9 of the two papers. Each graph is given by listing the new triples in that graph that were not in the previous triples.

### 1.1 Notation

In addition to the abbreviations specified in [2], we have the following: disjoint With, intersect ectionOf, equivalentClass, minCardinality, maxCardinality, cardinality, differentFrom, AllDifferent, distinctMembers, FunctionalProperty, subClass of, allValuesFrom, someValuesFrom, ContainerMembershipProperty, versionInfo. and ^^xsd:boolean.

Some of the triples in the first few sections (5 to 9), are shown with a wavy line under, such as  $("0"^{\wedge}b,type,eg:c)$ . This means that the triple is not included in the graphs  $H_i$ , i.e. sections 10 to 22.

To avoid too much redundancy we also use the following sets of nodes, identified by their first element, with an overline.

Relating to classes			
$b_{10}$	$b_{10}, b_9,$		
Nothing	Nothing, $b_2$ , DataRange, DataProp, DeprClass, DeprProp,		
	Ontology, Restrict, Alt, Bag, Seq, Statement, XMLLiteral,		
	Container,		
$\overline{b_3}$	$b_3, b_6, b_7,$		
owl:Class	owl:Class, rdfs:Class,		
ObjProp	ObjProp, Prop,		
Thing	Thing, Resource,		
Relating to properties			
$b_{11}$	$b_{11}, b_9,$		
priorVers	priorVers, allVals, backCompat, card, hasValue, imports,		
-	incompat, maxCard, minCard, onProperty, someVals, versInfo, _1,		
	object, predicate, subject, value, comment, isDefinedBy, label,		
	member, seeAlso,		

These are used in *metatriples* like (owl:Class, equivC, owl:Class) in the listing. This one stands for four triples in the graph, by taking each member of the set for both subject and object. Some of these may have occurred earlier in the construction.

### 2 The initial graph

This is given in section 5. It has several features intended to illustrate the action of the construction on literals, which while somewhat artificial in this example, are part of the D-semantics [3], and covered by our method.

- We use non-canonical lexical forms. The notion of 'canonical' used in  $\mathcal{VL}_D$  is not, in general, the same as that in XML Schema [4], which provides several different 'canonical' forms for the same value depending on which derived datatype is used. So in general, the literal replacement step between  $G_4$  and  $H_0$  is necessary. In our example, it is artificial.
- We use URI and blank nodes which implicitly represent literals; and then use some of these in predicate position.

### 3 The Function $\psi$

The function  $\psi$ , see section 9.2 of [2], is determined by some D-interpretation of  $G_4$ . We take  $\psi$  to be:

$$\psi(x) = \begin{cases} "\texttt{true}"^{\wedge}\texttt{b} & x = "\texttt{1}"^{\wedge}\texttt{b} \\ "\texttt{false}"^{\wedge}\texttt{b} & x = "\texttt{0}"^{\wedge}\texttt{b} \\ "\texttt{true}"^{\wedge}\texttt{b} & x = \texttt{eg:v} \\ "\texttt{false}"^{\wedge}\texttt{b} & x = b_{15} \\ x & \text{otherwise} \end{cases}$$
(1)

Others would be found from different D-interpretations.

### 4 The Interpretation

The interpretation in section 10 of [2], uses the graph presented in this document, along with the empty sting, the boolean datatype, and its values as the domain of discourse. The function  $\chi$  is

then given as:

$$\chi(x) = \begin{cases} \text{the boolean datatype} & x = \texttt{boolean} \\ \texttt{""} & x = \texttt{""} \\ \text{TRUE} & x = \texttt{"true"}^{\wedge \wedge}\texttt{b} \\ \text{FALSE} & x = \texttt{"false"}^{\wedge \wedge}\texttt{b} \\ x & \text{otherwise} \end{cases}$$
(2)

With the last line meaning the occurrences of the appropriate text string in triples (and implicitly in metatriples) in sections 5 to 22, except where deleted by the wavy line. The actual interpretation is built as specified in [2], via the function  $\theta$ , which we modify, following ter Horst [5] to deal with the rdf:\_*i*:

$$\theta(x) = \begin{cases} \chi(-1) & x \in \{ \mathtt{rdf} : i : i = 2, 3 \dots \} \\ \chi(x) & x \in \mathrm{nd}(H_{12}) \\ \chi(\psi(x)) & x \in \mathrm{nd}(G_4) \end{cases}$$
(3)

#### $\mathbf{5}$ $G_0$

("",type,eg:c)	(eg:a, eg:p, eg:v)	(eg:p, range, boolean)
("0"^^b,type,eg:c)	$(\texttt{eg:b},\texttt{eg:p},b_{15})$	
(eg:a,eg:p,"1"^^b)	(eg:v,eg:v,eg:v)	

#### 6 $G_1$

(boolean, type, Datatype)	$("false"^{\wedge}b,type,boolean)$
("",type,Literal)	$("true"^{\wedge\wedge}b, type, boolean)$

#### $G_2$ 7

(nil, type, List) (XMLLiteral, type, Datatype) (\_1, type, Prop) (\_1, domain, Resource) (first, type, Prop) (first, domain, List) (object, type, Prop) (object, domain, Statement) (predicate, type, Prop) (predicate, domain, Statement) (rest, type, Prop) (rest, domain, List) (subject, type, Prop) (subject, domain, Statement) (type, type, Prop) (type, domain, Resource) (value, type, Prop) (value, domain, Resource)  $(\_1, type, CMemProp)$ (comment, domain, Resource)  $({\tt object}, {\tt range}, {\tt Resource})$  $({\tt isDefinedBy}, {\tt range}, {\tt Resource})$ (Bag, subClass, Container) (predicate, range, Resource) (label, range, Literal) (rest, range, List) (member, range, Resource) (subject, range, Resource) (range, range, rdfs:Class) (type, range, rdfs:Class) (seeAlso, range, Resource) (value, range, Resource) (subClass, range, rdfs:Class) (comment, range, Literal) (subPropOf, range, Prop) (Alt, subClass, Container) (domain, range, rdfs:Class)

(domain,domain,Prop)(isDefinedBy, domain, Resource) (label, domain, Resource) (member, domain, Resource) (range, domain, Prop) (seeAlso, domain, Resource) (subClass, domain, rdfs:Class) (subPropOf, domain, Prop) (\_1, range, Resource) (first, range, Resource)

(Seq, subClass, Container) (XMLLiteral, subClass, Literal) (CMemProp, subClass, Prop)  $({\tt Datatype}, {\tt subClass}, {\tt rdfs:Class})$ (isDefinedBy, subPropOf, seeAlso)

# **8** $G_3$

(eg:p,type,Prop) (eg:v,type,Prop) (domain,type,Prop) (range,type,Prop) (subClass,type,Prop) (subPropOf,type,Prop) ("",type,Resource) ("0^^^ b,type,Resource) ("1"^^ b,type,Resource) ("false"^^ b,type,Resource)

(subject, type, Resource) (type, type, Resource) (value, type, Resource) (rdfs:Class, type, Resource) (Container, type, Resource) (CMemProp, type, Resource) (Datatype, type, Resource)

# **9** $G_4$

(comment, type, Prop) (isDefinedBy, type, Prop) (label, type, Prop) (member, type, Prop) (seeAlso, type, Prop) (eg:c, type, rdfs:Class) (Alt, type, rdfs:Class) (Bag, type, rdfs:Class) (List, type, rdfs:Class) (Prop, type, rdfs:Class) ("true"^^b, type, Resource) (b15.type, Resource) (eg:a, type, Resource) (eg:b, type, Resource) (eg:c, type, Resource) (eg:v, type, Resource) (eg:v, type, Resource) (Alt, type, Resource) (List, type, Resource)

(Literal, type, Resource) (Resource, type, Resource) (comment, type, Resource) (domain, type, Resource) (isDefinedBy, type, Resource) (label, type, Resource) (member, type, Resource) (Prop, type, Resource) (Seq, type, Resource) (Statement, type, Resource) (XMLLiteral, type, Resource) (\_1, type, Resource) (first, type, Resource) (nil, type, Resource) (object, type, Resource) (predicate, type, Resource) (rest, type, Resource)

(range,type,Resource) (seeAlso,type,Resource) (subClass,type,Resource) (subPropOf,type,Resource) (boolean,type,Resource)

(Seq,type,rdfs:Class) (Statement,type,rdfs:Class) (XMLLiteral,type,rdfs:Class) (rdfs:Class,type,rdfs:Class) (Container,type,rdfs:Class) (CMemProp,type,rdfs:Class) (Datatype,type,rdfs:Class) (Literal,type,rdfs:Class) (Resource,type,rdfs:Class) (boolean,type,rdfs:Class) ("1"^^b, type, Literal) ("false"^^b, type, Literal) ("true"^^b, type, Literal) (b15, type, Literal) (eg:v, type, Literal) ("1"^^b, type, boolean) (b15, type, boolean) (b15, type, boolean) (eg:v, type, boolean) (boolean, subClass, Literal) (-1, subPropOf, member)

### **10** *H*<sub>0</sub>

 ("false"^^ b, type, eg:c)
 ("true"^^ b, "true"^^ b, "true"^^ b)
 (eg:b, eg:p, "false"^^ b)

 ("true"^^ b, type, Prop)
 (eg:a, eg:p, "true"^^ b)

### **11** $H_1$

(priorVers,type,FunProp) (priorVers,type,InvFunProp) (backCompat,type,OntProp) (imports,type,OntProp) (incompat,type,OntProp) (priorVers,type,OntProp) (equivProp,type,SymProp) (inverseOf,type,SymProp) (subClass,type,TransProp) (subPropOf,type,TransProp) (allVals,type,Prop) (card,type,Prop) (complmntOf,type,Prop) (different,type,Prop) (disjoint,type,Prop) (dstnctMems,type,Prop) (hasValue,type,Prop) (intersect,type,Prop) (inverseOf,type,Prop) (maxCard,type,Prop) (minCard, type, Prop) (onProperty, type, Prop) (oneOf, type, Prop) (someVals, type, Prop) (unionOf, type, Prop)  $(b_2, type, rdfs:Class)$ (owl:Class,type,rdfs:Class) (DataRange, type, rdfs:Class) (DeprClass, type, rdfs:Class) (DeprProp, type, rdfs:Class)

 $(b_8, \texttt{equivProp}, b_8)$ (incompat, equivProp, backCompat) (priorVers, equivProp, backCompat) (inverseOf, inverseOf, inverseOf) (priorVers, equivProp, priorVers)  $(b_3, \texttt{intersect}, b_5)$  $(\overline{\texttt{owl:Class}}, \texttt{intersect}, b_2)$  $(rdfs:Class, intersect, b_3)$  $(b_9, \texttt{inverseOf}, b_{12})$  $(b_{12}, \texttt{inverseOf}, b_9)$ (equivProp, inverseOf, equivProp)

 $(b_4, \texttt{rest}, \texttt{nil})$  $(b_5, rest, nil)$  $(b_9, \texttt{domain}, b_{10})$  $(b_{10}, \texttt{domain}, b_8)$  $(b_{12}, \texttt{domain}, b_{10})$  $(b_9, range, b_{10})$ 

 $({\tt Nothing}, {\tt type}, {\tt rdfs:Class})$ (Ontology, type, rdfs:Class) (Literal, type, Datatype)  $(b_2, b_8, b_4)$  $(b_3, b_8, b_2)$  $(b_8, b_8, b_2)$  $(b_8, b_8, b_3)$  $(b_2, b_9, b_3)$  $(b_4, b_{10}, b_5)$  $(b_2, b_{11}, b_3)$ 

(imports, inverseOf, priorVers) (incompat, inverseOf, backCompat) (priorVers, inverseOf, backCompat) (priorVers, inverseOf, priorVers)  $(b_2, \texttt{oneOf}, \texttt{nil})$  $(b_3, \texttt{oneOf}, b_2)$  $(b_6, \texttt{oneOf}, b_2)$  $(b_6, \texttt{oneOf}, b_3)$  $(b_7, \texttt{oneOf}, b_3)$ 

 $(b_{10}, range, b_8)$ (AnnProp, subClass, Prop) (DataProp, subClass, Prop) (OntProp, subClass, Prop) (Restrict, subClass, rdfs:Class) (Prop, subClass, ObjProp)

### $(b_3, b_{12}, b_2)$

```
(Nothing, complmntOf, Thing)
(Thing, complmntOf, Nothing)
(Thing, different, Nothing)
(Nothing, disjoint, Nothing)
(Nothing, disjoint, Resource)
(Resource, disjoint, Nothing)
(b_1, \texttt{dstnctMems}, b_2)
(b_1, \texttt{dstnctMems}, b_3)
(Thing, equivC, Thing)
```

(Thing, sameAs, Thing)  $(b_2, \texttt{unionOf}, \texttt{nil})$  $(\overline{\texttt{owl:Class}}, \texttt{unionOf}, b_2)$  $(rdfs:Class,unionOf,b_3)$  $(b_2, first, rdfs: Class)$  $(b_3, first, rdfs:Class)$  $(b_4, first, Nothing)$  $(b_5, first, b_3)$  $(b_2, \texttt{rest}, \texttt{nil})$  $(b_3, \texttt{rest}, \texttt{nil})$ 

(Prop, subClass, Thing) (rdfs:Class, subClass, owl:Class) (Resource, subClass, Thing)  $(b_8, \mathtt{subPropOf}, b_8)$ (inverseOf, subPropOf, inverseOf) (priorVers, subPropOf, b<sub>9</sub>)

### 12 $H_2$

 $(b_4, \mathtt{type}, b_8)$  $(b_5, type, b_8)$  $(b_2, type, b_{10})$  $(b_3, type, b_{10})$  $(\overline{b_3}, \texttt{type}, \overline{\texttt{owl:Class}})$  $(b_8, type, \overline{owl:Class})$  $(\overline{b_{10}}, \texttt{type}, \overline{\texttt{owl:Class}})$ (eg:c,type,owl:Class) (AllDiff, type, owl:Class) (AnnProp, type, owl:Class) (different, type, ObjProp) (disjoint, type, ObjProp) (dstnctMems, type, ObjProp) (intersect, type, ObjProp) (inverseOf, type, ObjProp) (oneOf, type, ObjProp) (priorVers, type, ObjProp) (unionOf,type,ObjProp) (first, type, ObjProp) (rest, type, ObjProp)

(owl:Class, type, owl:Class) (FunProp, type, owl:Class) (InvFunProp, type, owl:Class) (Nothing, type, owl:Class) (ObjProp, type, owl:Class) (OntProp, type, owl:Class) (SymProp, type, owl:Class) (Thing, type, owl:Class) (TransProp, type, owl:Class) (List, type, owl:Class)

(type, type, ObjProp) (domain,type,ObjProp) (range,type,ObjProp) (subClass,type,ObjProp) (subPropOf, type, ObjProp) ("", type, Thing)  $(\texttt{"false"}^{\wedge\wedge}\texttt{b},\texttt{type},\texttt{Thing})$ ("true"^^b, type, Thing)  $(b_1, type, \overline{Thing})$  $(\overline{b_3}, \mathtt{type}, \overline{\mathtt{Thing}})$ 

(CMemProp, type, owl:Class) (Datatype, type, owl:Class) (Literal, type, owl:Class) (boolean, type, owl:Class) ("true"^^b, type, ObjProp)  $(b_8, \mathtt{type}, \overline{\mathtt{ObjProp}})$  $(\overline{b_{10}}, \texttt{type}, \overline{\texttt{ObjProp}})$  $(b_{12}, \texttt{type}, \overline{\texttt{ObjProp}})$ (eg:p,type,ObjProp) (complmntOf, type, ObjProp)

 $(b_4, type, \overline{Thing})$  $(b_5, type, \overline{Thing})$  $(b_8, \mathtt{type}, \overline{\mathtt{Thing}})$  $(\overline{b_{10}}, \mathtt{type}, \overline{\mathtt{Thing}})$  $(b_{12}, type, \overline{Thing})$ (eg:a, type, Thing) (eg:b, type, Thing) (eg:c,type,Thing) (eg:p,type,Thing) (AllDiff, type, Thing) (AnnProp, type, Thing) (owl:Class, type, Thing) (FunProp, type, Thing) (InvFunProp, type, Thing) (Nothing, type, Thing) (ObjProp, type, Thing) (OntProp, type, Thing) (SymProp, type, Thing) (Thing, type, Thing) (TransProp, type, Thing) (subClass, type, Thing) (subPropOf, type, Thing)

(subPropOf,type,Thing) (boolean,type,Thing)

### **13** *H*<sub>3</sub>

(b11, type, Prop) (equivC, type, Prop) (equivProp, type, Prop)

## **14** $H_4$

 $("true"^{\wedge \wedge} b, domain, Resource) \\ (b_8, domain, Resource) \\ (b_{10}, domain, Resource) \\ (\overline{b_{11}}, domain, Resource) \\ (b_{12}, domain, Resource) \\ (eg:p, domain, Resource) \\ (complmntOf, domain, Resource) \\ (different, domain, Resource) \\ (disjoint, domain, Resource) \\ (dstnctMems, dstnctMems, dstnct$ 

(priorVers,domain,List) (priorVers,domain,CMemProp) (priorVers,domain,Datatype) (priorVers,domain,Literal) (priorVers,domain,Resource) (priorVers,domain,Resource) (unionOf,domain,Resource) (first,domain,Resource) (rest,domain,Resource)

 $(priorVers, range, b_8)$   $(priorVers, range, \overline{b_{10}})$  (priorVers, range, eg:c) (priorVers, range, AllDiff) (priorVers, range, AnnProp)  $(priorVers, range, \overline{owl:Class})$  (priorVers, range, FunProp) (priorVers, range, InvFunProp)  $(priorVers, range, \overline{Nothing})$   $(priorVers, range, \overline{ObjProp})$ 

(complmntOf, type, Thing) (different, type, Thing) (disjoint, type, Thing) (dstnctMems, type, Thing) (dstnctMems, type, Thing) (equivProp, type, Thing) (intersect, type, Thing) (inverseOf, type, Thing) (oneOf, type, Thing) (priorVers, type, Thing) (unionOf, type, Thing)

 $egin{aligned} (b_2, \texttt{type}, \texttt{List}) \ (b_3, \texttt{type}, \texttt{List}) \ (b_4, \texttt{type}, \texttt{List}) \end{aligned}$ 

(sameAs,type,Prop) (b<sub>11</sub>,type,Resource) (equivC,type,Resource) (List, type, Thing) (first, type, Thing) (nil, type, Thing) (rest, type, Thing) (type, type, Thing) (CMemProp, type, Thing) (Datatype, type, Thing) (Literal, type, Thing) (domain, type, Thing) (range, type, Thing) (b5, type, List)

(Literal, subClass, Literal)

(sameAs, type, Resource)

 $\begin{array}{l} (\texttt{equivC},\texttt{domain},\texttt{Resource})\\ (\texttt{equivProp},\texttt{domain},\texttt{Resource})\\ (\texttt{intersect},\texttt{domain},\texttt{Resource})\\ (\texttt{interseOf},\texttt{domain},\texttt{Resource})\\ (\texttt{oneOf},\texttt{domain},\texttt{Resource})\\ (\texttt{oneOf},\texttt{domain},\texttt{Resource})\\ (\texttt{priorVers},\texttt{domain},\overline{b_3})\\ (\texttt{priorVers},\texttt{domain},\overline{b_{10}})\\ (\texttt{priorVers},\texttt{domain},\overline{b_{10}})\\ (\texttt{priorVers},\texttt{domain},\texttt{eg:c})\\ (\texttt{priorVers},\texttt{domain},\texttt{AllDiff}) \end{array}$ 

 $\begin{array}{l} (\text{domain}, \text{domain}, \text{Resource}) \\ (\text{range}, \text{domain}, \text{Resource}) \\ (\text{subClass}, \text{domain}, \text{Resource}) \\ (\text{subPropOf}, \text{domain}, \text{Resource}) \\ ("\text{true"}^{\wedge \wedge} \text{b}, \text{range}, \text{Resource}) \\ (b_8, \text{range}, \text{Resource}) \\ (b_{10}, \text{range}, \text{Resource}) \\ (b_{11}, \text{range}, \text{Resource}) \\ (b_{12}, \text{range}, \text{Resource}) \\ (\text{eg:p, range}, \text{Resource}) \\ \end{array}$ 

(priorVers,range,OntProp) (priorVers,range,SymProp) (priorVers,range,Thing) (priorVers,range,TransProp) (priorVers,range,List) (priorVers,range,CMemProp) (priorVers,range,Datatype) (priorVers,range,Literal) (priorVers,range,Resource) (priorVers,range,boolean)

(priorVers, domain, AnnProp) (priorVers, domain, owl:Class) (priorVers, domain, FunProp) (priorVers, domain, InvFunProp) (priorVers, domain, Nothing) (priorVers, domain, ObjProp) (priorVers, domain, OntProp) (priorVers, domain, SymProp) (priorVers, domain, Thing) (priorVers, domain, TransProp) (complmntOf, range, Resource) (different, range, Resource) (disjoint, range, Resource) (dstnctMems, range, Resource) (equivC, range, Resource) (equivProp, range, Resource) (intersect, range, Resource) (inverseOf, range, Resource) (oneOf, range, Resource)  $(priorVers, range, \overline{b_3})$ (sameAs, range, Resource) (unionOf, range, Resource) (rest, range, Resource) (type, range, Resource)

(domain, range, Resource) (range, range, Resource) (subClass, range, Resource) (subPropOf, range, Resource) (b<sub>2</sub>, subClass, b<sub>2</sub>) (b<sub>3</sub>, subClass, b<sub>3</sub>)

```
(b_6, \texttt{subClass}, b_6)
                                       (DataProp, subClass, DataProp)
                                                                               (SymProp, subClass, SymProp)
(b_7, \mathtt{subClass}, b_7)
                                       (DeprClass, subClass, DeprClass)
                                                                               (Thing, subClass, Thing)
(b_8, \mathtt{subClass}, b_8)
                                       (DeprProp, subClass, DeprProp)
                                                                               (TransProp, subClass, TransProp)
(b_9, \texttt{subClass}, b_9)
                                       (FunProp, subClass, FunProp)
                                                                               (Alt, subClass, Alt)
(b_{10}, \texttt{subClass}, b_{10})
                                                                               (Bag, subClass, Bag)
                                       (InvFunProp, subClass, InvFunProp)
(eg:c,subClass,eg:c)
                                       (Nothing, subClass, Nothing)
                                                                               (List, subClass, List)
(AllDiff, subClass, AllDiff)
                                       (ObjProp, subClass, ObjProp)
                                                                               (Prop, subClass, Prop)
(AnnProp, subClass, AnnProp)
                                       (Ontology, subClass, Ontology)
                                                                               (Seq, subClass, Seq)
(owl:Class, subClass, owl:Class)
                                       (OntProp, subClass, OntProp)
                                                                               (Statement, subClass, Statement)
(DataRange, subClass, DataRange)
                                       (Restrict, subClass, Restrict)
                                                                               (XMLLiteral, subClass, XMLLiteral)
(rdfs:Class, subClass, rdfs:Class)
                                       (b_{12}, \mathtt{subPropOf}, b_{12})
                                                                               (equivProp, subPropOf, equivProp)
(Container, subClass, Container)
                                       (eg:p,subPropOf,eg:p)
                                                                               (hasValue, subPropOf, hasValue)
                                       (allVals, subPropOf, allVals)
                                                                               (imports, subPropOf, imports)
(CMemProp, subClass, CMemProp)
(Datatype, subClass, Datatype)
                                       (backCompat, subPropOf, backCompat) (incompat, subPropOf, incompat)
(Resource, subClass, Resource)
                                       (card, subPropOf, card)
                                                                               (intersect, subPropOf, intersect)
(boolean, subClass, boolean)
                                       (complmntOf, subPropOf, complmntOf) (maxCard, subPropOf, maxCard)
("true"^{\wedge\wedge}b, subPropOf, "true"^{\wedge\wedge}b)
                                       (different, subPropOf, different)
                                                                               (minCard, subPropOf, minCard)
(b_9, \mathtt{subPropOf}, b_9)
                                       (disjoint, subPropOf, disjoint)
                                                                               (onProperty, subPropOf, onProperty)
(b_{10}, \mathtt{subPropOf}, b_{10})
                                       (dstnctMems, subPropOf, dstnctMems) (oneOf, subPropOf, oneOf)
(b_{11}, \mathtt{subPropOf}, b_{11})
                                       (equivC, subPropOf, equivC)
                                                                               (priorVers, subPropOf, priorVers)
(sameAs, subPropOf, sameAs)
                                       (predicate, subPropOf, predicate)
                                                                               (isDefinedBy, subPropOf, isDefinedBy)
(someVals, subPropOf, someVals)
                                       (rest, subPropOf, rest)
                                                                               (label, subPropOf, label)
(unionOf, subPropOf, unionOf)
                                       (subject, subPropOf, subject)
                                                                               (member, subPropOf, member)
(versInfo, subPropOf, versInfo)
                                       (type, subPropOf, type)
                                                                               (range, subPropOf, range)
(\_1, \texttt{subPropOf}, \_1)
                                       (value, subPropOf, value)
                                                                               (seeAlso, subPropOf, seeAlso)
                                       (comment, subPropOf, comment)
(first.subPropOf.first)
```

```
(subClass, subPropOf, subClass)
(subPropOf, subPropOf, subPropOf)
```

## **15** $H_5$

(b11,type,ObjProp) (equivC,type,ObjProp) (equivProp,type,ObjProp)

(object, subPropOf, object)

(sameAs, type, ObjProp) (b<sub>11</sub>, type, Thing) (equivC, type, Thing)

(domain, subPropOf, domain)

(sameAs, type, Thing)

### **16** $H_6$

The nodes of the graph  $H_5$  are: { "", "false"^^b, "true"^^b,  $b_1$ ,  $b_2$ ,  $b_3$ ,  $b_4$ ,  $b_5$ ,  $b_6$ ,  $b_7$ ,  $b_8$ ,  $b_9$ ,  $b_{10}$ ,  $b_{11}$ ,  $b_{12}$ , eg:a, eg:b, eg:c, eg:p, AllDiff, AnnProp, owl:Class, DataRange, DataProp, DeprClass, DeprProp, FunProp, InvFunProp, Nothing, ObjProp, Ontology, OntProp, Restrict, SymProp, Thing, TransProp, allVals, backCompat, card, complmntOf, different, disjoint, dstnctMems, equivC, equivProp, hasValue, imports, incompat, intersect, inverseOf, maxCard, minCard, onProperty, oneOf, priorVers, sameAs, someVals, unionOf, versInfo, Alt, Bag, List, Prop, Seq, Statement, XMLLiteral, \_1, first, nil, object, predicate, rest, subject, type, value, rdfs:Class, Container, CMemProp, Datatype, Literal, Resource, comment, domain, isDefinedBy, label, member, range, seeAlso, subClass, subPropOf, boolean, }.

For any single node n in this set, we add  $(n, \mathtt{sameAs}, n)$ .

For any pair of distinct nodes n, n' in this set, we add (n, different, n').

## **17** $H_7$

$("true"^{\wedge\wedge}b,type,FunProp)$
$(b_{10}, {\tt type}, {\tt FunProp})$
$(\overline{b_{11}}, {\tt type}, {\tt FunProp})$
$(b_{12}, {\tt type}, {\tt FunProp})$
(eg:p,type,FunProp)
$(\overline{\texttt{priorVers}}, \texttt{type}, \texttt{FunProp})$
(sameAs, type, FunProp)
(first, type, FunProp)
(rest,type,FunProp)
$("true"^{\wedge\wedge}b, type, InvFunProp)$

(equivC, type, TransProp) (equivProp, type, TransProp) (priorVers, type, TransProp) (b10, type, InvFunProp) (b11, type, InvFunProp) (b12, type, InvFunProp) (eg:p, type, InvFunProp) (dstnctMems, type, InvFunProp) (priorVers, type, InvFunProp) (sameAs, type, InvFunProp) ("true"^^b, type, SymProp) (complmntOf, type, SymProp) (different, type, SymProp)

(sameAs, type, TransProp) (rest, type, TransProp) (eg:c, type, Datatype)  $\begin{array}{l} (\texttt{disjoint, type, SymProp}) \\ (\texttt{equivC, type, SymProp}) \\ (\overline{\texttt{priorVers}}, \texttt{type, SymProp}) \\ (\texttt{sameAs, type, SymProp}) \\ (\texttt{"true"}^{\wedge\wedge}\texttt{b}, \texttt{type, TransProp}) \\ (b_{10}, \texttt{type, TransProp}) \\ (\overline{b_{11}}, \texttt{type, TransProp}) \\ (b_{12}, \texttt{type, TransProp}) \\ (\texttt{eg:p, type, TransProp}) \\ (\texttt{dstnctMems, type, TransProp}) \end{array}$ 

(eg:c, subClass, Literal)

## **18** $H_8$

(Nothing, complmntOf, Thing) (Thing, complmntOf, Nothing)  $(\overline{b_3}, \texttt{disjoint}, b_8)$  $(\overline{b_3}, \texttt{disjoint}, \overline{b_{10}})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{eg:c})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{AllDiff})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{AnnProp})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{FunProp})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{InvFunProp})$  $(\overline{b_3}, \texttt{disjoint}, \overline{\texttt{Nothing}})$  $(b_8, \texttt{disjoint}, \texttt{CMemProp})$  $(b_8, \texttt{disjoint}, \texttt{Datatype})$  $(b_8, \texttt{disjoint}, \texttt{Literal})$  $(b_8, \texttt{disjoint}, \texttt{boolean})$  $(\overline{b_{10}}, \texttt{disjoint}, \overline{b_3})$  $(\overline{b_{10}}, \texttt{disjoint}, \texttt{eg:c})$  $(\overline{b_{10}}, \texttt{disjoint}, \texttt{AllDiff})$  $(\overline{b_{10}}, \texttt{disjoint}, \texttt{AnnProp})$  $(\overline{b_{10}}, \texttt{disjoint}, \texttt{FunProp})$  $(\overline{b_{10}}, \texttt{disjoint}, \texttt{InvFunProp})$ (eg:c,disjoint,SymProp) (eg:c,disjoint,TransProp) (eg:c,disjoint,List) (eg:c,disjoint,CMemProp) (eg:c,disjoint,Datatype)  $(AllDiff, disjoint, \overline{b_3})$  $(\texttt{AllDiff}, \texttt{disjoint}, b_8)$  $(AllDiff, disjoint, \overline{b_{10}})$ (AllDiff, disjoint, eg:c) (AllDiff, disjoint, AnnProp)  $(\overline{b_3}, \texttt{disjoint}, \overline{\texttt{ObjProp}})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{OntProp})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{OntProp})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{SymProp})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{List})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{CMemProp})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{Datatype})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{Literal})$  $(\overline{b_3}, \texttt{disjoint}, \texttt{boolean})$  $(b_8, \texttt{disjoint}, \overline{b_3})$ 

 $\begin{array}{l} (\overline{b_{10}}, \texttt{disjoint}, \overline{\texttt{Nothing}}) \\ (\overline{b_{10}}, \texttt{disjoint}, \overline{\texttt{ObjProp}}) \\ (\overline{b_{10}}, \texttt{disjoint}, \texttt{OntProp}) \\ (\overline{b_{10}}, \texttt{disjoint}, \texttt{OntProp}) \\ (\overline{b_{10}}, \texttt{disjoint}, \mathbb{SymProp}) \\ (\overline{b_{10}}, \texttt{disjoint}, \mathsf{TransProp}) \\ (\overline{b_{10}}, \texttt{disjoint}, \texttt{CMemProp}) \\ (\overline{b_{10}}, \texttt{disjoint}, \texttt{Datatype}) \\ (\overline{b_{10}}, \texttt{disjoint}, \texttt{Literal}) \\ (\overline{b_{10}}, \texttt{disjoint}, \texttt{boolean}) \\ (\texttt{eg:c, disjoint}, \overline{b_3}) \end{array}$ 

(AllDiff,disjoint, owl:Class) (AllDiff,disjoint,FunProp) (AllDiff,disjoint,InvFunProp) (AllDiff,disjoint,Nothing) (AllDiff,disjoint,ObjProp) (AllDiff,disjoint,OntProp) (AllDiff,disjoint,SymProp) (AllDiff,disjoint,TransProp) (AllDiff,disjoint,List) (AllDiff,disjoint,CMemProp)  $(b_8, \texttt{disjoint}, \texttt{eg:c}) \\ (b_8, \texttt{disjoint}, \texttt{AllDiff}) \\ (b_8, \texttt{disjoint}, \texttt{AnnProp}) \\ (b_8, \texttt{disjoint}, \texttt{FunProp}) \\ (b_8, \texttt{disjoint}, \texttt{InvFunProp}) \\ (b_8, \texttt{disjoint}, \texttt{Nothing}) \\ (b_8, \texttt{disjoint}, \texttt{Nothing}) \\ (b_8, \texttt{disjoint}, \texttt{ObjProp}) \\ (b_8, \texttt{disjoint}, \texttt{OntProp}) \\ (b_8, \texttt{disjoint}, \texttt{SymProp}) \\ (b_8, \texttt{disjoint}, \texttt{TransProp}) \\ (b_8, \texttt{disjoint}, \texttt{TransProp}) \\$ 

 $\begin{array}{l} (\texttt{eg:c,disjoint},b_8) \\ (\texttt{eg:c,disjoint},\overline{b_{10}}) \\ (\texttt{eg:c,disjoint},\texttt{AllDiff}) \\ (\texttt{eg:c,disjoint},\texttt{AnnProp}) \\ (\texttt{eg:c,disjoint},\overline{\texttt{owl:Class}}) \\ (\texttt{eg:c,disjoint},\texttt{FunProp}) \\ (\texttt{eg:c,disjoint},\texttt{InvFunProp}) \\ (\texttt{eg:c,disjoint},\overline{\texttt{Nothing}}) \\ (\texttt{eg:c,disjoint},\overline{\texttt{ObjProp}}) \\ (\texttt{eg:c,disjoint},\texttt{OntProp}) \\ (\texttt{eg:c,disjoint},\texttt{OntProp}) \end{array}$ 

 $\begin{array}{l} (\texttt{AllDiff}, \texttt{disjoint}, \texttt{Datatype}) \\ (\texttt{AllDiff}, \texttt{disjoint}, \texttt{Literal}) \\ (\texttt{AllDiff}, \texttt{disjoint}, \texttt{boolean}) \\ (\texttt{AnnProp}, \texttt{disjoint}, \overline{b_3}) \\ (\texttt{AnnProp}, \texttt{disjoint}, \overline{b_{10}}) \\ (\texttt{AnnProp}, \texttt{disjoint}, \overline{b_{10}}) \\ (\texttt{AnnProp}, \texttt{disjoint}, \texttt{eg:c}) \\ (\texttt{AnnProp}, \texttt{disjoint}, \texttt{allDiff}) \\ (\texttt{AnnProp}, \texttt{disjoint}, \overline{\texttt{owl:Class}}) \\ (\texttt{AnnProp}, \texttt{disjoint}, \overline{\texttt{Nothing}}) \end{array}$ 

(AnnProp, disjoint, OntProp) (AnnProp, disjoint, List) (AnnProp, disjoint, CMemProp) (AnnProp, disjoint, Datatype) (AnnProp, disjoint, Literal) (AnnProp, disjoint, boolean) (owl:Class, disjoint, eg:c) (owl:Class, disjoint, AllDiff) (owl:Class, disjoint, AnnProp) (owl:Class, disjoint, Nothing)

 $\begin{array}{l} (\operatorname{InvFunProp, disjoint, Datatype}) \\ (\overline{\operatorname{Nothing}, \operatorname{disjoint}, \overline{b_3}}) \\ (\overline{\operatorname{Nothing}, \operatorname{disjoint}, \overline{b_3}}) \\ (\overline{\operatorname{Nothing}, \operatorname{disjoint}, \overline{b_{10}}}) \\ (\overline{\operatorname{Nothing}, \operatorname{disjoint}, \operatorname{eg:c}}) \\ (\overline{\operatorname{Nothing}, \operatorname{disjoint}, \operatorname{AllDiff}}) \\ (\overline{\operatorname{Nothing}, \operatorname{disjoint}, \operatorname{AnnProp}}) \\ (\overline{\operatorname{Nothing}, \operatorname{disjoint}, \overline{\operatorname{owl:Class}}}) \\ (\overline{\operatorname{Nothing}, \operatorname{disjoint}, \overline{\operatorname{runProp}}}) \\ (\overline{\operatorname{Nothing}, \operatorname{disjoint}, \operatorname{FunProp}}) \\ (\overline{\operatorname{Nothing}, \operatorname{disjoint}, \operatorname{InvFunProp}}) \end{array}$ 

(OntProp, disjoint, b<sub>8</sub>) (OntProp, disjoint, b<sub>10</sub>) (OntProp, disjoint, eg:c) (OntProp, disjoint, AllDiff) (OntProp, disjoint, AnnProp) (OntProp, disjoint, owl:Class) (OntProp, disjoint, Nothing) (OntProp, disjoint, List) (OntProp, disjoint, CMemProp) (OntProp, disjoint, Datatype)

 $\begin{array}{l} (\texttt{Datatype, disjoint, }\overline{b_{10}}) \\ (\texttt{Datatype, disjoint, eg:c}) \\ (\texttt{Datatype, disjoint, AllDiff}) \\ (\texttt{Datatype, disjoint, AnnProp}) \\ (\texttt{Datatype, disjoint, FunProp}) \\ (\texttt{Datatype, disjoint, InvFunProp}) \\ (\texttt{Datatype, disjoint, }\overline{\mathsf{Nothing}}) \\ (\texttt{Datatype, disjoint, }\overline{\mathsf{ObjProp}}) \\ (\texttt{Datatype, disjoint, } \mathsf{OntProp}) \\ (\texttt{Datatype, disjoint, } \mathsf{OntProp}) \\ (\texttt{Datatype, disjoint, } \mathsf{SymProp}) \\ (\texttt{Datatype, disjoint, } \mathsf{SymProp}) \end{array}$ 

 $\begin{array}{l} (\overline{\texttt{owl:Class}},\texttt{disjoint},\texttt{OntProp})\\ (\overline{\texttt{owl:Class}},\texttt{disjoint},\texttt{SymProp})\\ (\overline{\texttt{owl:Class}},\texttt{disjoint},\texttt{CMemProp})\\ (\overline{\texttt{owl:Class}},\texttt{disjoint},\texttt{CMemProp})\\ (\overline{\texttt{owl:Class}},\texttt{disjoint},\texttt{Literal})\\ (\overline{\texttt{owl:Class}},\texttt{disjoint},\texttt{boolean})\\ (\overline{\texttt{FunProp}},\texttt{disjoint},\overline{b_3})\\ (\overline{\texttt{FunProp}},\texttt{disjoint},\overline{b_3})\\ (\overline{\texttt{FunProp}},\texttt{disjoint},\overline{b_{10}})\\ (\overline{\texttt{FunProp}},\texttt{disjoint},eg:c)\\ (\overline{\texttt{FunProp}},\texttt{disjoint},\texttt{AllDiff}) \end{array}$ 

(Nothing, disjoint, Nothing) (Nothing, disjoint, ObjProp) (Nothing, disjoint, OntProp) (Nothing, disjoint, SymProp) (Nothing, disjoint, TransProp) (Nothing, disjoint, List) (Nothing, disjoint, CMemProp) (Nothing, disjoint, Datatype) (Nothing, disjoint, Literal)

 $\begin{array}{l} (\texttt{OntProp, disjoint, Literal}) \\ (\texttt{OntProp, disjoint, boolean}) \\ (\texttt{SymProp, disjoint, } \overline{b_3}) \\ (\texttt{SymProp, disjoint, } b_8) \\ (\texttt{SymProp, disjoint, } g:c) \\ (\texttt{SymProp, disjoint, eg:c}) \\ (\texttt{SymProp, disjoint, allDiff}) \\ (\texttt{SymProp, disjoint, } \overline{\texttt{owl:Class}}) \\ (\texttt{SymProp, disjoint, } \overline{\texttt{Nothing}}) \\ (\texttt{SymProp, disjoint, List}) \end{array}$ 

 $\begin{array}{l} (\texttt{Datatype, disjoint, TransProp})\\ (\texttt{Datatype, disjoint, List})\\ (\texttt{Datatype, disjoint, CMemProp})\\ (\texttt{Datatype, disjoint, Literal})\\ (\texttt{Datatype, disjoint, boolean})\\ (\texttt{Literal, disjoint, } \overline{b_3})\\ (\texttt{Literal, disjoint, } \overline{b_{10}})\\ (\texttt{Literal, disjoint, } \overline{b_{10}})\\ (\texttt{Literal, disjoint, AllDiff})\\ (\texttt{Literal, disjoint, AnnProp})\end{array}$ 

 $\begin{array}{l} ({\tt FunProp, disjoint, \overline{Nothing}}) \\ ({\tt FunProp, disjoint, List}) \\ ({\tt FunProp, disjoint, Datatype}) \\ ({\tt InvFunProp, disjoint, \overline{b_3}}) \\ ({\tt InvFunProp, disjoint, b_8}) \\ ({\tt InvFunProp, disjoint, \overline{b_{10}}}) \\ ({\tt InvFunProp, disjoint, eg:c}) \\ ({\tt InvFunProp, disjoint, AllDiff}) \\ ({\tt InvFunProp, disjoint, \overline{Nothing}}) \\ ({\tt InvFunProp, disjoint, List}) \end{array}$ 

 $\begin{array}{l} (\overline{\text{Nothing}}, \operatorname{disjoint}, \operatorname{boolean}) \\ (\overline{\text{ObjProp}}, \operatorname{disjoint}, \overline{b_3}) \\ (\overline{\text{ObjProp}}, \operatorname{disjoint}, \overline{b_3}) \\ (\overline{\text{ObjProp}}, \operatorname{disjoint}, \overline{b_{10}}) \\ (\overline{\text{ObjProp}}, \operatorname{disjoint}, \operatorname{eg:c}) \\ (\overline{\text{ObjProp}}, \operatorname{disjoint}, \operatorname{AllDiff}) \\ (\overline{\text{ObjProp}}, \operatorname{disjoint}, \overline{\text{Nothing}}) \\ (\overline{\text{ObjProp}}, \operatorname{disjoint}, \operatorname{List}) \\ (\overline{\text{ObjProp}}, \operatorname{disjoint}, \operatorname{Datatype}) \\ (\overline{\text{OntProp}}, \operatorname{disjoint}, \overline{b_3}) \end{array}$ 

 $\begin{array}{l} (\texttt{SymProp},\texttt{disjoint},\texttt{Datatype})\\ (\overline{\texttt{Thing}},\texttt{disjoint},\overline{\texttt{Nothing}})\\ (\overline{\texttt{TransProp}},\texttt{disjoint},\overline{b_3})\\ (\overline{\texttt{TransProp}},\texttt{disjoint},\overline{b_8})\\ (\overline{\texttt{TransProp}},\texttt{disjoint},\overline{b_{10}})\\ (\overline{\texttt{TransProp}},\texttt{disjoint},\texttt{eg:c})\\ (\overline{\texttt{TransProp}},\texttt{disjoint},\texttt{AllDiff})\\ (\overline{\texttt{TransProp}},\texttt{disjoint},\overline{\texttt{Nothing}})\\ (\overline{\texttt{TransProp}},\texttt{disjoint},\overline{\texttt{List}})\\ (\overline{\texttt{TransProp}},\texttt{disjoint},\texttt{List})\\ (\overline{\texttt{TransProp}},\texttt{disjoint},\texttt{Datatype})\end{array}$ 

 $\begin{array}{l} (\texttt{CMemProp, disjoint, AnnProp}) \\ (\texttt{CMemProp, disjoint, \overline{owl:Class}}) \\ (\texttt{CMemProp, disjoint, \overline{Nothing}}) \\ (\texttt{CMemProp, disjoint, OntProp}) \\ (\texttt{CMemProp, disjoint, List}) \\ (\texttt{CMemProp, disjoint, Datatype}) \\ (\texttt{CMemProp, disjoint, Literal}) \\ (\texttt{CMemProp, disjoint, boolean}) \\ (\texttt{Datatype, disjoint, } b_8) \end{array}$ 

```
\begin{array}{l} (\texttt{boolean},\texttt{disjoint},\texttt{AnnProp})\\ (\texttt{boolean},\texttt{disjoint},\overline{\texttt{owl:Class}})\\ (\texttt{boolean},\texttt{disjoint},\overline{\texttt{owl:Class}})\\ (\texttt{boolean},\texttt{disjoint},\texttt{OntProp})\\ (\texttt{boolean},\texttt{disjoint},\texttt{List})\\ (\texttt{boolean},\texttt{disjoint},\texttt{List})\\ (\texttt{boolean},\texttt{disjoint},\texttt{CMemProp})\\ (\texttt{boolean},\texttt{disjoint},\texttt{Datatype})\\ (\overline{b_3},\texttt{equivC},\overline{b_3})\\ (\overline{b_{10}},\texttt{equivC},\overline{b_{10}})\\ \end{array}
```

```
 \begin{array}{l} (b_8, \texttt{subClass}, \texttt{List}) \\ (b_9, \texttt{subClass}, b_{10}) \\ (b_{10}, \texttt{subClass}, b_9) \\ (\overline{b_{10}}, \texttt{subClass}, \overline{\texttt{owl:Class}}) \\ (\overline{b_{10}}, \texttt{subClass}, \overline{\texttt{owl:Class}}) \\ (\overline{b_{10}}, \texttt{subClass}, \overline{\texttt{Thing}}) \\ (\overline{b_{10}}, \texttt{subClass}, \texttt{List}) \\ (\texttt{eg:c}, \texttt{subClass}, \overline{\texttt{Thing}}) \\ (\texttt{AllDiff}, \texttt{subClass}, \overline{\texttt{Thing}}) \\ (\texttt{AnnProp}, \texttt{subClass}, \texttt{FunProp}) \\ (\texttt{AnnProp}, \texttt{subClass}, \texttt{InvFunProp}) \end{array}
```

```
(Nothing, subClass, Nothing)
(Nothing, subClass, ObjProp)
(Nothing, subClass, OntProp)
(Nothing, subClass, SymProp)
(Nothing, subClass, SymProp)
(Nothing, subClass, TransProp)
(Nothing, subClass, List)
(Nothing, subClass, CMemProp)
(Nothing, subClass, Datatype)
(Nothing, subClass, Literal)
```

```
(CMemProp, subClass, TransProp)
(Datatype, subClass, owl:Class)
```

(eg:c, equivC, eg:c) (AllDiff, equivC, AllDiff) (AnnProp, equivC, AnnProp) (owl:Class, equivC, owl:Class) (FunProp, equivC, FunProp) (InvFunProp, equivC, InvFunProp) (Nothing, equivC, Nothing) (ObjProp, equivC, ObjProp) (OntProp, equivC, OntProp) (SymProp, equivC, SymProp)

(AnnProp, subClass, ObjProp) (AnnProp, subClass, SymProp) (AnnProp, subClass, Thing) (AnnProp, subClass, TransProp) (owl:Class, subClass, Thing) (owl:Class, subClass, Thing) (owl:Class, subClass, rdfs:Class) (FunProp, subClass, ObjProp) (FunProp, subClass, Thing) (InvFunProp, subClass, Thing) (InvFunProp, subClass, Thing)

(Nothing, subClass, boolean) (ObjProp, subClass, Thing) (ObjProp, subClass, Prop) (OntProp, subClass, FunProp) (OntProp, subClass, InvFunProp) (OntProp, subClass, ObjProp) (OntProp, subClass, SymProp) (OntProp, subClass, Thing) (OntProp, subClass, TransProp) (SymProp, subClass, ObjProp)

(Datatype, subClass, Thing) (Literal, subClass, Thing)  $\begin{array}{l} ({\tt TransProp}, {\tt equivC}, {\tt TransProp}) \\ ({\tt List}, {\tt equivC}, {\tt List}) \\ ({\tt CMemProp}, {\tt equivC}, {\tt CMemProp}) \\ ({\tt Datatype}, {\tt equivC}, {\tt Datatype}) \\ ({\tt Literal}, {\tt equivC}, {\tt Datatype}) \\ ({\tt Literal}, {\tt equivC}, {\tt Literal}) \\ ({\tt boolean}, {\tt equivC}, {\tt boolean}) \\ (\overline{b_3}, {\tt subClass}, \overline{b_3}) \\ (\overline{b_3}, {\tt subClass}, \overline{{\tt out:Class}}) \\ (\overline{b_3}, {\tt subClass}, \overline{{\tt Thing}}) \\ (b_8, {\tt subClass}, \overline{{\tt Thing}}) \end{array}$ 

 $\begin{array}{l} ({\tt InvFunProp, subClass, TransProp})\\ \hline ({\tt Nothing, subClass, } \overline{b_3})\\ \hline ({\tt Nothing, subClass, } \overline{b_3})\\ \hline ({\tt Nothing, subClass, } \overline{b_{10}})\\ \hline ({\tt Nothing, subClass, eg:c})\\ \hline ({\tt Nothing, subClass, AllDiff})\\ \hline ({\tt Nothing, subClass, AnnProp})\\ \hline ({\tt Nothing, subClass, } \overline{owl:Class})\\ \hline ({\tt Nothing, subClass, FunProp})\\ \hline ({\tt Nothing, subClass, FunProp})\\ \hline ({\tt Nothing, subClass, InvFunProp})\\ \hline \end{array}$ 

(SymProp, subClass, Thing) (Thing, subClass, Resource) (TransProp, subClass, ObjProp) (TransProp, subClass, Thing) (List, subClass, Thing) (CMemProp, subClass, FunProp) (CMemProp, subClass, InvFunProp) (CMemProp, subClass, ObjProp) (CMemProp, subClass, SymProp) (CMemProp, subClass, Thing)

(boolean, subClass, Thing)

### **19** *H*<sub>9</sub>

$(b_1, \mathtt{dstnctMems}, b_4)$
$(b_1, \mathtt{dstnctMems}, b_5)$
$(b_1, {\tt dstnctMems}, {\tt nil})$
$(\overline{b_3}, \texttt{intersect}, b_5)$
$(\texttt{owl:Class},\texttt{intersect},b_3)$

 $(\overline{\text{Nothing}}, \text{intersect}, b_4)$  $(\overline{\text{Thing}}, \text{intersect}, \text{nil})$  $(b_3, \text{oneOf}, b_3)$  $(b_7, \text{oneOf}, b_2)$  $(\overline{\text{Nothing}}, \text{oneOf}, \text{nil})$ 

 $(\overline{b_3}, \texttt{unionOf}, b_5)$ (owl:Class, unionOf,  $b_3$ ) (Nothing, unionOf,  $b_4$ ) (Nothing, unionOf, nil)

### **20** $H_{10}$

$("true"^{\wedge}b, inverseOf, "true"^{\wedge}b)$	$({\tt complmntOf}, {\tt inverseOf}, {\tt complmntOf})$	$(\verb"equivC", inverseOf", equivC")$
$(b_{11}, \texttt{inverseOf}, b_{12})$	$({\tt different}, {\tt inverseOf}, {\tt different})$	$(\overline{\texttt{priorVers}}, \texttt{inverseOf}, \overline{\texttt{priorVers}})$
$(b_{12}, \texttt{inverseOf}, b_{11})$	$({\tt disjoint}, {\tt inverseOf}, {\tt disjoint})$	$({\tt sameAs}, {\tt inverseOf}, {\tt sameAs})$

# **21** $H_{11}$

("true"^^b, domain, FunProp) ("true"^^b, domain, InvFunProp) ("true"^^b, domain, ObjProp) ("true"^^b, domain, SymProp) ("true"^^b, domain, TransProp) ("true"^^b, domain, Literal) ("true"^^b, domain, Literal) ("true"^^b, domain, boolean) (b<sub>8</sub>, domain, Owl:Class) (b<sub>8</sub>, domain, Thing)

(equivC, domain, Thing) (equivProp, domain, 0bjProp) (equivProp, domain, Thing) (intersect, domain, owl:Class) (intersect, domain, Thing) (inverse0f, domain, 0bjProp) (inverse0f, domain, Thing) (one0f, domain, owl:Class) (one0f, domain, Thing) (priorVers, domain, b<sub>3</sub>)

(unionOf, domain, owl:Class) (unionOf, domain, Thing) (first, domain, Thing) (rest, domain, Thing) (type, domain, Thing) (domain, domain, ObjProp) (domain, domain, ObjProp) (range, domain, Thing) (subClass, domain, owl:Class)

(b12, range, b10) (b12, range, owl:Class) (b12, range, Thing) (b12, range, List) (eg:p, range, List) (eg:p, range, Literal) (complmntOf, range, owl:Class) (complmntOf, range, Thing) (different, range, owl:Class)

(priorVers, range, FunProp) (priorVers, range, InvFunProp) (priorVers, range, InvFunProp) (priorVers, range, ObjProp) (priorVers, range, OntProp) (priorVers, range, SymProp) (priorVers, range, Thing) (priorVers, range, TransProp) (priorVers, range, List) (priorVers, range, CMemProp) 
$$\label{eq:priorVers} \begin{split} & (\overline{\texttt{priorVers}}, \texttt{domain}, b_8) \\ & (\overline{\texttt{priorVers}}, \texttt{domain}, \overline{b_{10}}) \\ & (\overline{\texttt{priorVers}}, \texttt{domain}, \texttt{eg:c}) \\ & (\overline{\texttt{priorVers}}, \texttt{domain}, \texttt{AllDiff}) \\ & (\overline{\texttt{priorVers}}, \texttt{domain}, \texttt{AnnProp}) \\ & (\overline{\texttt{priorVers}}, \texttt{domain}, \overline{\texttt{owl:Class}}) \\ & (\overline{\texttt{priorVers}}, \texttt{domain}, \overline{\texttt{FunProp}}) \\ & (\overline{\texttt{priorVers}}, \texttt{domain}, \mathbb{InvFunProp}) \\ & (\overline{\texttt{priorVers}}, \texttt{domain}, \mathbb{Nothing}) \\ & (\overline{\texttt{priorVers}}, \texttt{domain}, \overline{\texttt{ObjProp}}) \end{split}$$

(subClass,domain,Thing) (subPropOf,domain,ObjProp) (subPropOf,domain,Thing) ("true"^^b,range,FunProp) ("true"^^b,range,InvFunProp) ("true"^^b,range,ObjProp) ("true"^^b,range,SymProp) ("true"^b,range,Thing) ("true"^b,range,TransProp) ("true"^b,range,Literal)

(disjoint,range,Thing) (dstnctMems,range,Thing) (dstnctMems,range,List) (equivC,range,oul:Class) (equivC,range,Thing) (equivProp,range,ObjProp) (equivProp,range,Thing) (intersect,range,Thing) (intersect,range,List) (inverseOf,range,ObjProp)

(priorVers, range, Datatype) (priorVers, range, Literal) (priorVers, range, Literal) (sameAs, range, Thing) (unionOf, range, Thing) (unionOf, range, List) (first, range, owl:Class) (first, range, Thing) (rest, range, Thing) (type, range, owl:Class) (b12, domain, List) (eg:p, domain, Thing) (complmntOf, domain, owl:Class) (complmntOf, domain, Thing) (different, domain, Thing) (disjoint, domain, owl:Class) (disjoint, domain, AllDiff) (dstnctMems, domain, AllDiff) (dstnctMems, domain, Thing) (equivC, domain, owl:Class)

(priorVers, domain, OntProp) (priorVers, domain, SymProp) (priorVers, domain, Thing) (priorVers, domain, TransProp) (priorVers, domain, List) (priorVers, domain, CMemProp) (priorVers, domain, Datatype) (priorVers, domain, Literal) (priorVers, domain, boolean) (sameAs, domain, Thing)

 $("true"^{\wedge}b, range, boolean)$  $(b_8, range, Thing)$  $(b_8, range, List)$  $(b_{10}, range, List)$  $(b_{10}, range, List)$  $(\overline{b_{10}}, range, \overline{b_{10}})$  $(\overline{b_{11}}, range, \overline{b_{10}})$  $(\overline{b_{11}}, range, \overline{owl:Class})$  $(\overline{b_{11}}, range, Thing)$  $(\overline{b_{11}}, range, List)$  $(b_{12}, range, b_8)$ 

 $\begin{array}{l} (\texttt{inverseOf}, \texttt{range}, \texttt{Thing}) \\ (\texttt{oneOf}, \texttt{range}, \texttt{Thing}) \\ (\texttt{oneOf}, \texttt{range}, \texttt{List}) \\ (\overline{\texttt{priorVers}}, \texttt{range}, \overline{b_3}) \\ (\overline{\texttt{priorVers}}, \texttt{range}, \overline{b_3}) \\ (\overline{\texttt{priorVers}}, \texttt{range}, \overline{b_{10}}) \\ (\overline{\texttt{priorVers}}, \texttt{range}, \texttt{eg:c}) \\ (\overline{\texttt{priorVers}}, \texttt{range}, \texttt{AllDiff}) \\ (\overline{\texttt{priorVers}}, \texttt{range}, \texttt{AnnProp}) \\ (\overline{\texttt{priorVers}}, \texttt{range}, \overline{\texttt{oyl:Class}}) \end{array}$ 

(type, range, Thing) (domain, range, owl:Class) (domain, range, Thing) (range, range, owl:Class) (range, range, Thing) (subClass, range, owl:Class) (subClass, range, Thing) (subPropOf, range, ObjProp) (subPropOf, range, Thing)

## **22** $H_{12}$

$("true"^{\wedge}b, equivProp, "true"^{\wedge}b)$	$(\verb"equivProp, equivProp, equivProp")$	(domain, equivProp, domain)
$(b_{10},  textsf{equivProp}, b_{10})$	$({\tt intersect}, {\tt equivProp}, {\tt intersect})$	$(\tt range, \tt equivProp, range)$
$(\overline{b_{11}},  extsf{equivProp}, \overline{b_{11}})$	$({\tt inverseOf}, {\tt equivProp}, {\tt inverseOf})$	$({\tt subClass}, {\tt equivProp}, {\tt subClass})$
$(b_{12},  textsf{equivProp}, b_{12})$	(oneOf,equivProp,oneOf)	$({\tt subPropOf}, {\tt equivProp}, {\tt subPropOf})$
(eg:p,equivProp,eg:p)	$(\overline{\texttt{priorVers}}, \texttt{equivProp}, \overline{\texttt{priorVers}})$	$(\texttt{"true"}^{\wedge\wedge}\texttt{b}, \texttt{subPropOf}, \texttt{equivProp})$
(complmntOf, equivProp, complmntOf)	$({\tt sameAs}, {\tt equivProp}, {\tt sameAs})$	$(\texttt{"true"}^{\wedge\wedge}\texttt{b}, \texttt{subPropOf}, \texttt{inverseOf})$
(different, equivProp, different)	$({\tt unionOf}, {\tt equivProp}, {\tt unionOf})$	$(\texttt{"true"}^{\wedge\wedge}b, \texttt{subPropOf}, \texttt{sameAs})$
(disjoint, equivProp, disjoint)	(first, equivProp, first)	$("true"^{\wedge\wedge}b, subPropOf, subPropOf)$
(dstnctMems, equivProp, dstnctMems)	(rest, equivProp, rest)	$(b_8, {\tt subPropOf}, {\tt different})$
(equivC, equivProp, equivC)	(type,equivProp,type)	$(b_9, {\tt subPropOf}, b_{11})$
$(b_{10}, {\tt subPropOf}, {\tt different})$	(complmntOf, subPropOf, different)	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, b_{12})$
$(b_{11}, \mathtt{subPropOf}, b_9)$	$({\tt complmntOf}, {\tt subPropOf}, {\tt disjoint})$	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{eg:p})$
$(\overline{b_{11}}, {\tt subPropOf}, {\tt different})$	$({\tt dstnctMems}, {\tt subPropOf}, {\tt different})$	(priorVers, subPropOf, complmntOf)
$(\overline{b_{11}}, {\tt subPropOf}, {\tt disjoint})$	$(\verb"equivC, subPropOf, subClass")$	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{different})$
$(\overline{b_{11}}, {\tt subPropOf}, {\tt subClass})$	$(\verb"equivProp, subPropOf, subPropOf")$	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{disjoint})$
$(b_{12}, \mathtt{subPropOf}, b_8)$	$({\tt intersect}, {\tt subPropOf}, {\tt different})$	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{dstnctMems})$
$(b_{12}, {\tt subPropOf}, {\tt different})$	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{"true"}^{\wedge \wedge}\texttt{b})$	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{equivC})$
$(b_{12}, {\tt subPropOf}, {\tt disjoint})$	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, b_8)$	$(\overline{\tt priorVers}, {\tt subPropOf}, {\tt equivProp})$
$(b_{12}, {\tt subPropOf}, {\tt oneOf})$	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, b_{10})$	$(\overline{\tt priorVers}, {\tt subPropOf}, {\tt intersect})$
(eg:p,subPropOf,different)	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \overline{b_{11}})$	$(\overline{\tt priorVers}, \tt subPropOf, inverseOf)$
$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{oneOf})$	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{rest})$	$(\overline{\tt priorVers}, {\tt subPropOf}, {\tt subPropOf})$
$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \overline{\texttt{priorVers}})$	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{type})$	$({\tt unionOf}, {\tt subPropOf}, {\tt different})$
$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{sameAs})$	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{domain})$	$(\tt first, \tt subPropOf, \tt different)$
$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{unionOf})$	$(\overline{\texttt{priorVers}}, \texttt{subPropOf}, \texttt{range})$	$({\tt rest}, {\tt subPropOf}, {\tt different})$
(priorVers, subPropOf, first)	(priorVers, subPropOf, subClass)	

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